Rev. version May 1997

CCE/97-3

A CONSTRUCTIONAL APPROACH TO THE INTERNATIONAL SYSTEM OF UNITS (SI)

Ulrich Feller

Swiss Federal Office of Metrology, CH-3084 Wabern, Switzerland

Abstract

To communicate the magnitude of quantities, a system of units is needed. It allows the attribution of a defined value to every quantity which describes a particular physical phenomenon.

Every system of units is based on a few conventional assignments, in which a quantity with a fixed value is assigned to a well-defined physical phenomenon. The number of assignments is given by the necessity for fixing quantitatively all physical quantities, without introducing any overdetermination in the process. The assigned values have no uncertainty, as they are not measured. All other values of a quantity are traceable to the conventionally assigned values. The International System of units (SI) is based on five conventional assignments, from which all units, except the kilogram, are derived.

The constructional view adopted in this paper points out the functional dependence among the base units and shows explicitly the role fundamental constants play in the construction of the SI. It turns out that well-known statements encountered in official publications on the SI are ambiguous or even in contradiction with physical facts.

1. Introduction

The definition of appropriate physical quantities and their units of measurement is not given by nature. This is a subject of science and evolves with the growing insight in our physical environment. It was a considerable progress for science, trade, and teaching when in the second half of this century the many puzzling and concurring systems of units could be reduced worldwide to a single system of units, the well known Système International d'Unités (SI). Since then, the exchange of information on measurement results has become much easier, as (nearly) everybody around the world is using the same units and confusing conversions from one system of units to another are no longer necessary.

The evolution of the SI has to be understood as a historical process that has not only been governed by scientific considerations. Practical, political, subjective and other motives have influenced the evolution as well and have left traces in the definitions and in the structure of the SI. A recent review on this subject with numerous references is given in [1]. Conceptually, the historical roots sometimes rather obscure the true structure of the SI. In actual publications formulations are still used that do not reflect the physical facts.

Originally, the units were grouped into three classes: base units, derived units and supplementary units [2]. At the 20th Conférence Générale des Poids et Mesures (CGPM) of 1995, the class of supplementary units was abolished. The set of SI units consists now of two classes, the base units and the derived units.

The two remaining classes do not coincide with the functional relations between the units, and formulations used in relevant publications veil the existing dependencies among the base units instead of explaining them. In [2] for example, it is stated that the SI is based "…on a choice of seven well-defined units which, by convention, are regarded as dimensionally independent". In another publication [3], the corresponding wording reads: "Table 1 gives the seven base quantities, assumed to be mutually independent, on which the SI is founded; and the names and symbols of their respective units, called SI base units". In [4], the base units are introduced as follows (translated from German): "The SI … is founded on seven base units, with which, in principle, all physical quantities can be measured. The number of base units is determined by the number of physical quantities in the system which are considered as independent". The International Vocabulary of Basic and General Terms in Metrology [5] defines a base unit as a unit of measurement of a base quantity. A base quantity in turn is defined as one of the quantities that, in a system of quantities, is conventionally accepted as functionally independent of the others.

Formulations of that kind raise questions: Are the base units dependent or independent of each other in the mathematical sense and according to physical laws? Are all the SI base units necessary in order to make all physical quantities measurable?

Another question concerns the dimensionality of a system of units. It is related to the number of base units. Traditionally, cgs-systems in mechanics and electrodynamics[1] were called three-dimensional systems (based on cm, g, s), whereas the MKSA subset of the SI was called a four-dimensional system of units (based on m, kg, s, A). Group theoretical considerations were made to justify this designation [6]. In physics and mathematics, the number of dimensions is usually defined in terms of a mathematical structure (e.g. vector space, or representation of a group). It is natural to ask about the structure behind a system of units.

Another problem deserves attention: It is interesting to note that fundamental constants are *not* mentioned explicitly in the definitions of the SI base units, although some of them are based on fundamental constants. What is the role of these fundamental constants in the SI?

It is the aim of the paper to point out some shortcomings in actual presentations of the SI and to promote a representation which highlights explicitly the physical situation. A full account of the structure of the SI can only be given if the relations among the base units and among the base units and the fundamental constants are clearly stated.

2. Terminology

In order to make clear what exactly is meant by some of the terms used in the subsequent discussion, a few preliminary explanations are necessary.

The terms “quantity” and “unit” are often used alternately and the phenomenon described by the quantity and the quantity itself are often not clearly distinguished. The term “mass”

[1] Examples of cgs-systems: Electrostatic, Electromagnetic, Gaussian, Heaviside-Lorentz system of units.

may illustrate this: Firstly, “mass” is an abstract physical quantity used in the framework of a theory. Secondly, the term “mass” is used when a particular property of a body is designated, and thirdly, “mass” is sometimes even used when the body itself is addressed. However, in the discussion of systems of units and quantities it is necessary to distinguish clearly between the different meanings. In contrast to [5], the term “quantity” will be used in this paper only in the general sense, i.e., a quantity is considered as an abstract concept defined implicitly in the framework of a physical theory. The definition of a quantity is given by all the statements which can be made about it, including all the equations or laws linking the quantity to other quantities.

Quantities are used to describe special aspects of observable phenomena. In this process, they are associated with the phenomena. A well defined, particular phenomenon described or characterized by a quantity will be called a “realization” of the quantity.

For brevity, the term “phenomenon“ will be used in this paper as a general term for an observable fact or event which can be described by physical quantities.

The following examples may illustrate this terminology:
The term “length of a rod” means that the quantity “length” is used to characterize a special aspect of a particular rod, and “resistance of a wire” means that the quantity “resistance” is applied for characterizing a special electrical property of a particular wire. In these examples, the observable phenomena “rod” and “wire” are realizations of the quantities “length” and “resistance”, respectively.

3. Constructing a system of units

In order to communicate the magnitude of a quantity describing a particular phenomenon to other people, it is necessary that everybody knows how the magnitude is determined. Basically, an independent realization of the quantity can be selected, and this selection can be assigned by convention the value 1 of the quantity (a quantity is defined, like every mathematical variable, on a set of values it can assume). Such an assignment defines a unit. Obviously, the unit of a quantity consists of three parts: A well defined realization of the quantity, the number 1 and, optionally, a suffix designating the unit. Defined in this way, the unit has no uncertainty. The magnitude of the quantity describing any other realization can then be determined by comparing it with the unit. It is expressed as a multiple or fraction of the unit. As an example, the quantity “mass” is fixed quantitatively this way: The unit of mass consists of the international prototype of the kilogram, the number 1 and the designation “kilogram”.

In principle, each quantity could be determined quantitatively by selecting an independent realization of it, and by attributing the value 1 to this realization. Having done that, all physical quantities would be measurable, i.e., the magnitude of every quantity describing a particular phenomenon could be stated. Obviously, such a system of units would have a severe disadvantage. As the physical quantities are not all independent of each other, the value of a quantity would turn out to be different, when determined one time by comparison with its unit, or when determined another time by expressing the quantity as a function of other quantities. The system would be heavily overdetermined. In order to avoid different values for one and the same realization of the quantity, the number of independent units had to be reduced - by applying the physical laws - up to the point where all overdeterminations have been removed.

It is this picture of independent base units which is conveyed by dividing the units into so-called base units and derived units. This picture is further supported by formulations like those cited in the introduction. Unfortunately, the independent assignments necessary to fix the system of units and the artificial division in base and derived units do not go together in the SI. This is an unsatisfactory situation, as it hampers the understanding of the SI considerably. The concept of independent base units reflects the historical situation when all base units were defined in terms of a special artifact or a particular, independent process. Defined in such a way, base units were indeed independent of each other: A change of one base unit did not affect other base units.

The first reason why the SI does not correspond to the picture of a system based on a few, independent base units is that, except for the kilogram, no other base unit is realized this way.

The second reason is that not all SI-base units are independent of each other. Some of them are derived like the derived units of the SI, i.e., they can be expressed as a function of other base units. Therefore, it is misleading to use the term “derived units”, as it easily might be inferred that all the other units not called “derived units” are not derived, which would be wrong.

The question arises how the SI-system or any other system of units is fixed. The basic principles for the construction of a system of units are summarized below. Basically, there are two methods for fixing the values of a quantity: A direct or an indirect one.

Direct method:

- Select the quantity Q
- Select a well-defined realization of Q, independent of other quantities
- Assign it a value by convention (number and suffix, designating the unit of the quantity)

All other values of the quantity are determined by comparing the realizations directly - or over a chain of successive comparisons - with the realization associated with the assigned value. Obviously, the following statements apply:

- A quantity must have no more than one conventionally fixed value
- The numerical value needs not to be the number 1
- A value assigned by convention has no uncertainty, as it is not measured

As mentioned above, not all quantities must be fixed quantitatively by a conventional assignment, as this would introduce an overdetermination. So, most quantities must be fixed by the indirect method.

Indirect method:

- Using the known physical laws, the quantity Q is expressed as a function of other quantities Q_1, Q_2, Q_3, …, which are quantitatively already fixed: $Q = f(Q_1, Q_2, Q_3, \ldots)$.

Obviously, a quantity, expressed as a function of other quantitatively fixed quantities, cannot have its own realization with a conventionally assigned value.

The scientific purpose of a system of units is to define a consistent set of conventional assignments necessary for the measurement of all physical quantities, without introducing any overdetermination. Such a set forms the foundation of every system of units, as it allows to fix the value of every quantity in an unambiguous way. Unfortunately, actual presentations of the SI hardly give a full account of this most important aspect of a system of units.

It is important to note that, for a given state of established scientific knowledge, the number of quantities used to describe nature and the assumed valid laws linking them, are independent of the particular way quantities are measured. This means, that the number of conventional assignments is given, i.e., is independent of the particular system of units! To determine this number, in a arbitrary system of units only the number of independent values having no uncertainty must be counted (see paragraph 7).

Naturally, one is free to give some units an own name or to call them “base units” because of their fundamental importance in scientific or daily life. However, this linguistic aspect is metrologically of minor significance. Specifically, giving a unit its own name does not introduce any independence. This might sound trivial, but the opinion is widely spread that with the introduction of the unit ampere a new, independent unit had been introduced. However, dependencies are given by physical laws, and not by names of units.

In the following paragraphs, the SI base quantities will be analyzed with respect to the two methods described above.

4. Mass, length and time

As already mentioned above, mass is fixed quantitatively with a conventional assignment: The mass of the prototype of the kilogram has the assigned value of 1 kg by definition, therefore, its value has no uncertainty.

The quantity “length” is not fixed with a conventional assignment, there is no realization of the quantity “length” having a value without uncertainty. Conceptually, its values are derived from a time measurement using a universal law linking both quantities. The definition of the meter reads [2]:

The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.

The particular phenomenon used for this definition is the propagation of an electromagnetic wave in vacuum, and its velocity is assigned the fixed value of 299 792 458 m/s (no uncertainty!). With this definition, it is the quantity “velocity” which is fixed quantitatively by a conventional assignment. As the speed of light *c* has within the SI a constant value by definition, values of length can be determined by the law

$$\Delta l = c \cdot \Delta t . \qquad (1)$$

In this equation, length is expressed as a function of time and therefore, is a derived quantity. At the same time, length is in the SI a base quantity by definition. Conceptually this ambiguity is not satisfactory.

Whether the constant c is assigned the value 1, the value 299’792’458 or the value 299’792’458 m/s, is irrelevant with respect to the functional dependence between length and time. In the first two cases, values of length would be expressed in the same unit as time, in the third case the unit of length gets an own name. The choice is a pure matter of convenience and affects in no way the mathematical or physical dependence between the quantities “length” and “time”. For daily use, the third choice is the most convenient one, in high energy physics and relativity it is often the first one. Whether values of the quantity “length” are expressed in "meter" or in the same unit as time, is only a matter of convenience too. The abstract concept of the quantity “length” with all of its characteristic properties is neither affected by this choice nor by the way length is measured.

Expressing values of different quantities in the same unit is well known. The three space dimensions are all measured in m (although they are mathematically independent!). As a consequence, energy and torque are measured in the same unit, as well as other quantities [2, p. 74]. In practice, this causes no problem, as the qualities of different quantities are easily distinguished and not dependent on how they are measured.

The quantity length has always the same properties, whether its values are related to the former international prototype of the meter, or whether its values are derived from a time measurement. In the second case however, the value of length describing a particular phenomenon depends on how long a second is, in the first case not.

The discussion of the quantity “length” shows that in the SI the designations “base quantity” and “derived quantity” are ambiguous. From the conceptual point of view, it is unsatisfactory that the implicit assignment of a fixed value to the fundamental constant *c* in the definition of the meter is not clearly stated, because it is just this assignment which is essential for the quantitative determination of the physical quantities.

Analogous to the quantity “mass”, the values of the quantity “time” are fixed by a conventional assignment too. However, the numeric value assigned by convention is different from 1. The definition of the “second” reads: “The second is the duration of

9’192’631’770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom”, i.e.:

$$1\ \mathrm{s} = 9'192'631'770 \cdot T \ . \tag{2}$$

By this definition, the period *T* of the indicated radiation is assigned the value of $T = (1/9'192'631'770)\ \mathrm{s}$. This period is not measured, but fixed by convention. Thus, it has no uncertainty. A SI realization of the second must be derived from this definition and therefore, has an associated uncertainty.

5. Electrical quantities

The definition of adequate quantities and units in electricity has caused in the past considerable trouble [6, 7]. In publications, the reader often gets the impression that there are basic differences between the older cgs-systems and the SI. This impression is conveyed by presentations like the following:

- In older textbooks on electricity that not yet use SI units, electrical units in cgs-systems appear in half integer powers of g, cm, s, when expressed in base units.
- cgs-systems are described as dimensionally different from the MKSA subset of SI units. The cgs-systems are called three-dimensional systems, the MKSA-system is called a four-dimensional system of electromagnetism. There were even group theoretical efforts which made believe that there is an underlying mathematical structure supporting the difference in dimensionality [6].
- In [2], it is written that “… the system of quantities and the corresponding system of equations (of the cgs systems) are often different from those used with SI units”.

Theoretical literature on electromagnetism does not support this view. The classical textbook by Jackson [8] may be cited for many other textbooks. The reason is that the electromagnetic quantities based on cgs-systems as well as the corresponding quantities of the SI are both quantitatively derived from mechanical quantities, using either Coulomb’s or Ampere’s law (Figure 1). In all these systems, the same set of quantities is defined on the base of the same laws, these quantities describe the same electromagnetic phenomena and satisfy, up to some constants, the identical set of equations.[2] As a consequence, the so called "dimensionality", as used in the connection with a system of units, has no mathematical foundation.

As in the SI the constant $k_2 = \mu_0 / (4 \cdot \pi)$ (see Fig. 1) is a constant with a fixed value, current is, by Ampere's law, a function of the base quantities mass, length and time, i.e., current is a derived quantity. At the same time, the ampere is a base quantity by definition. There is the same, unsatisfactory ambiguity as with the quantity "length".

A particularly close similarity exists between the electromagnetic system and the SI: Their units differ only in powers of ten. The unit of current was defined to be exactly one-tenth of the unit of current in the electromagnetic system. In analogy to the relation among length and time, it is a pure matter of agreement whether the unit of current shall be given an own name or whether it shall be expressed in mechanical units. The quality of the quantity "current" is not affected by this purely practical aspect. In the electromagnetic system, it was decided to assign a pure number to the constant k_2, without any suffix, and to express the unit of current in g, cm, s. In the SI, it was decided to call the unit of current "ampere". Consequently, the dimension (N/A^2) was assigned to the constant k_2 to convert the dimension on the right side of Ampere's law back to the unit of force. It is easily shown that all dimensions (combinations of suffixes indicating the units involved) of quantities in the electromagnetic system of units and the SI are similar, when the substitution $\mathrm{a} := \mathrm{dyn}^{1/2} = \mathrm{g}^{1/2} \cdot \mathrm{cm}^{1/2} \cdot \mathrm{s}^{-1}$ is performed in the electromagnetic system (a: "ampere" in cgs-units, in analogy to A in the SI). Obviously, a substitution does not change a functional dependence!

Somebody not already familiar with electromagnetism might ask what the well known definition of the ampere has to do with all the basic equations in electricity. It is not clear too how a practical realization should look like using this definition. The answer to these questions is found in [9, p.133]: The only purpose of the definition was to fix the value of the free constant k_2 in Ampere's law, and to fix herewith the size of the unit, and not the methods to follow for a practical realization. The realization has to be effected according

---

[2] There are differences given in the scaling (g, cm, s ; kg, m, s; 4π) and in the way the propagation speed *c* of an electromagnetic wave is incorporated into the equations. The factor 4·π has historical roots: The concept of "field" was not available when these systems were developed. Therefore, the laws given above were written as long-distance-effect laws. As a consequence, the natural symmetry of fields generated by a point charge and a linear, current carrying wire was not expressed appropriately in the coordinates, which caused a scaling of the field quantities with the factor 4π.

to the general laws in electromagnetism. The definition of the ampere represents only a particular case of Ampere's general law, chosen for its simplicity.

There is nothing to say against a simple definition. But when the main purpose of a definition does not come out explicitly in the formulation, it does not serve its purpose! So, a presentation of the SI should clearly state that the definition serves to fix the value of the permeability of free space μ_0 , and that, by Ampere's law, current is a well defined function of force and length. This cross reference to electromagnetic theory and fundamental constants is necessary for the understanding of the dependencies among the units.

6. Temperature, amount of substance and luminous intensity

"The kelvin, unit of thermodynamic temperature, is the fraction 1/273,16 of the thermodynamic temperature of the triple point of water". By this definition, the temperature of the triple point of water is assigned the value of 273.16 K exactly. As is the case for the kilogram or the second, a change of any other unit does not affect the kelvin.

Conceptually, the mole, the unit of the amount of substance, is not necessary: For counting identical particles, no unit is necessary at all. The problem is, that in chemistry there are so many particles to count, that normal counting is not viable. It is much easier to do that approximately by weighing. The definition accounts for this problem: "The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12". The definition of the mole is an operational guide for determining approximately uncountable large numbers of identical particles. Doing this by a weighing process, the mole is linearly dependent on the kilogram: Cutting away half of the prototype of the kilogram, the mole would stand for only half of the number of particles too! It is important to realize that there is no new phenomenon which has an assigned value of one mole exactly.

The definition of the "candela" reads: "The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 x 10^{12} hertz and that has a radiant intensity in that direction of (1/683) watt per steradian". By definition, the candela is linearly dependent on the watt, the unit of power. There is no reason for not calling the candela a base unit, but it is wrong to speak of an assumed or considered, functional or dimensional independence upon the other base units. The photometric quantities are fixed by the radiometric quantities, weighted by the empirically determined sensitivity to light of the human eye.

The correct functional dependence between the base units is fundamental for the understanding of the SI system. Therefore, formulations which are in contradiction with physical facts like those cited in the introduction should be avoided. There is no reason not to call a certain set of units “base units”. But it should be clearly stated that a base unit is a base unit by agreement, and that the division in base and derived units does not reflect the functional dependence among units. So, it is pure convention that the SI base unit ampere is not expressed by mechanical units, in contrast to the unit of current in cgs-systems. But this is true for derived SI units as well: Who is aware that $kg \cdot m^2 \cdot A^{-2} \cdot s^{-3}$ is the SI suffix for the ohm, the unit of resistance?

7. Foundation of the SI

The scientifically essential part of a system of units is the selection of a consistent set of conventional assignments necessary for the measurement of all physical quantities (paragraph 3). It was shown in the previous paragraphs that in the SI the set of base units does not constitute such a consistent set: Firstly, because not all base units are necessary and secondly, because the base units, up to the kilogram, have no realization with an assigned value of 1, by convention. The question arises: Which are the conventional assignments fixing the SI?

Conceptually, there is no difference whether a fixed value is assigned to a quantity describing an aspect of an independent piece of material, e.g., the international prototype of the kilogram, or to a quantity describing an aspect of a radiation emitted and absorbed by a particular kind of atoms, or to a quantity describing an aspect of a propagating electromagnetic wave in vacuum. This fact is widely used in the construction of the SI. According to the discussion in the preceeding paragraphs, the conventional assignments of Table 1 can be identified.

With these assignments, all physical quantities become measurable and can be evaluated quantitatively. In particular, all SI units can be constructed starting from these five conventional assignments. By consequence, all values of a quantity describing a physical phenomenon are traceable to these five assigned values. The five assignments reflect the five basic observations of space, time, matter, charge and thermal phenomena. It appears that within the actual state of science five conventional assignments (arbitrary assignments of a fixed value to a quantity describing a well defined phenomenon) are necessary to fix quantitatively all the quantities used to describe the observable world. Of course, the number of independent assignments might change again in future when some further progress in the understanding of nature is achieved, e.g., the precise determination of the fine structure constant.

Whether the five assignments are made like in Table 1, or made for example by assigning ($h = c = k = \mu_0 = 1$, plus another consistent assignment), is of great practical - as it changes the system of units -, but not of conceptual importance. A pure matter of convenience is further the possibility to append a suffix to the assigned value identifying the unit of measurement. So, it is very practical when a value of a quantity can be recognized as belonging to length, mass, current, etc., and the correctness of a calculation can be checked by a dimensional analysis. Therefore, it is convenient to append a suffix to the assigned numeric values, e.g., m/s to c, N/A^2 to μ_0, etc. This identification, however, has nothing to do with the question of measurability or the dependence or independence of quantities, as the dependencies are given by the physical laws, not by the way units are called. Specifically, the introduction of the name “ampere” for the unit of current introduces neither an independence nor an additional degree of freedom in a mathematical sense.

In Table 1, the speed of light *c* and the permeability of vacuum μ_0 belong to the fundamental constants. An explicit integration of the fundamental constants in the definitions of the SI base units would certainly help for the understanding of the SI.

8. Units or fundamental constants ?

The five assignments of Table 1 are of very different quality:

The unit of mass is bound to a unique piece of material available at only one place in the world. Conceptually this is an easily comprehensible definition of a unit and its comparison with other realizations of mass is straight forward (intellectually, not metrologically!), but it represents the state of science at the end of the last century. Today, it can hardly satisfy the requirements to a base unit. Defined as a prototype, its availability is very limited and cannot be guaranteed in the long term. It is known to be unstable. The most important shortcoming is that the uncertainty of its stability is transferred to all quantities and fundamental constants related to the kilogram. These are not only many mechanical quantities but, through Ampere's law, all electrical quantities as well.

The particular radiation of Cs-133 and the triple point of water used for the definition of the second and the kelvin are in principle available everywhere, there is no spatial or temporal limitation. However, they are still associated with a complicated structure of matter.

By far the most universal phenomena are those linked with fundamental constants. Some of them are independent of any kind of complicated atomic or molecular structure. Conventional assignments based on these constants offer the highest security of temporal or spatial stability. Hoewver, it is more difficult to make use of such assignments in measurements. On the other hand, some fundamental constants appear in various physical laws and offer therefore the possibility to design more than one experiment for the measurement of a quantity. So, electrical quantities are today not related to a particular experiment, but every experiment based on Coulomb's law, Ampere's law or Gauss' law might serve for an absolute determination. Independent measurements offer a better possibility to detect systematic errors.

The shortcomings of the mass unit are well known and various experiments are in progress to replace the kilogram perhaps in the future [10]. The most universal approach to replace the kilogram would be the assignment of a value to Planck's constant h. This fundamental constant is realized in every elementary particle with spin $\neq 0$ and appears in most fundamental physical laws and many combinations of constants. Today, measurements for trade and industry would hardly suffer by such an assignment, the accuracy of scientific measurements on the atomic and subatomic level, however, could be improved considerably. By attributing a fixed value to the two constants h and c, the foundations of metrology and theory would again come closer to each other.

9. Conclusions

A system of units is used to express the magnitude of measured quantities. In the SI, all units are derived from a set of five conventional assignments: Fixed values are assigned to five quantities describing a well defined phenomenon. These five conventional assignments are independent of each other, they are not linked by physical laws. Actual presentations of the SI do hardly reflect this metrological foundation. There are even wordings which are in contradiction with the physical facts. The preceeding discussion can be summarized as follows:

- The classification in base and derived units does not reflect the functional dependence among the units. Any formulation suggesting a real, assumed, considered, dimensional or functional independence between base units is in contradiction with physical facts and should be avoided.
- The designation “derived unit” is misleading and should be avoided too, as some base units are derived as well.
- In the cgs-systems of units and the SI, the functional dependencies among the quantities are identical. The apparent difference in the dimension is an artifact of the SI and has no mathematical foundation. Group theoretical attempts suggesting a difference in dimension between the MKSA subset of the SI and cgs-systems are based on wrong assumptions [6].
- The key role of conventional assignments for the construction of a system of units should be explained and the fundamental constants fixed implicitly in the definitions of the meter and the ampere should be mentioned explicitly. The two conventional constants c and μ_0 are essential in the construction of the SI.

A full account of the structure of the SI can only be given if the relations among the base units and between the base units and the fundamental constants are clearly stated. Outdated formulations not in accordance with the actual structure should be avoided. Clear and unambiguous formulations would undoubtedly help in understanding and teaching the SI.

Acknowledgements

I am very indebted to my colleagues Marc Flüeli, Blaise Jeanneret , Beat Jeckelmann, Otto Piller and Wolfgang Schwitz for critical comments and stimulating discussions.

References

1. *Metrologia*, 1995, **31**, Number 6
2. *Le Système International d'Unités (SI)*, Sèvres (France), Bureau International des Poids et Mesures (BIPM), 1991.
3. Taylor B. N., *Guide for the use of the International System of Units (SI)*, NIST Spec. Publ. 811, Washington, U.S. Government Printing Office, 1995.
4. *Die gesetzlichen Masseinheiten in der Schweiz*, Wabern (Switzerland), Swiss Federal Office of Metrology, 1993.
5. *International Vocabulary of Basic and General Terms*, Geneva, ISO, 1993.
6. de Boer J., in [1], pp. 405-429.
7. Petley B.W., in [1], pp. 481-494, and pp. 495-502.
8. Jackson J.D., *Classical Electrodynamics*, New York, J. Wiley & Sons, 1965, p. 811.
9. *BIPM Proc.-Verb. Com. Int. Poids et Mesures*, 1946, **20**, p. 133.
10. Taylor B.N., *IEEE Trans. Instrum. Meas*., 1991, **40**, pp. 86-91.

Figure 1. Link of the different electrical systems of units to the mechanical units.

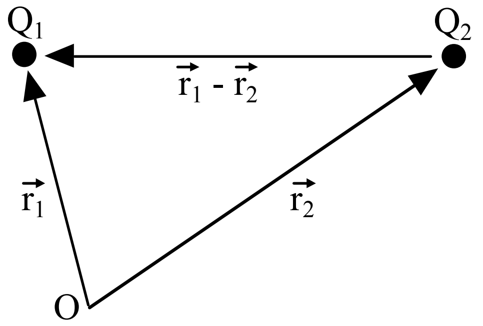
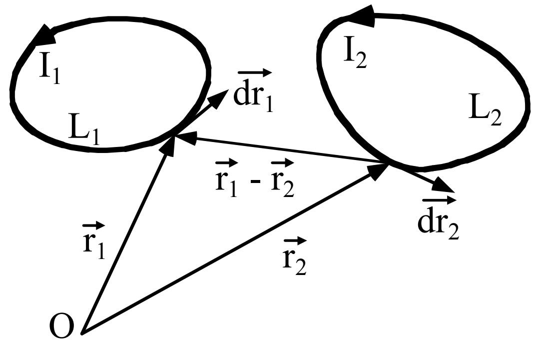
<table>
<tr><th>Coulomb's law</th><th>Ampere's law</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$\vec{K}_{12} = k_1 \cdot Q_1 \cdot Q_2 \cdot \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}$</td><td>$\vec{K}_{12} = k_2 \cdot I_1 \cdot I_2 \cdot \oint_{L_1}\oint_{L_2} \frac{d\vec{r}_1 \times [d\vec{r}_2 \times (\vec{r}_1 - \vec{r}_2)]}{|\vec{r}_1 - \vec{r}_2|^3}$</td></tr>
<tr><td>$k_1 = 1$: Electrostatic system of units, Gaussian system of units
$k_1 = 1/4\pi$: Heaviside-Lorentz system of units</td><td>$k_2 = 1$: Electromagnetic system of units
$k_2 = \mu_0 / (4 \cdot \pi)$: SI</td></tr>
</table>

Table 1. Conventional assignments, founding the SI.

Quantity	Realization of the quantity	Assigned value (exactly!)
Mass	International prototype of the kilogram	1 kg
Time	Period ot the radiation used for the derivation of the second	1/9'192'631'770 s
Velocity	Propagation speed of an electromagnetic wave in vacuum	299'792'458 m/s
Magnetic permeability	Magnetic permeability of empty space	$4 \cdot \pi \cdot 10^{-7}$ N/A^2
Temperature	Triple point of water	273,16 K