metrology of clocks & timescales

Laurent-Guy Bernier, METAS

summary (A)

- ✓ **introduction**
  - ☞ what is a clock, what is a timescale ?
  - ☞ international timescales TAI and UTC
  - ☞ Swiss timescales TA(CH) & UTC(CH)
- ✓ **examples of metrological problems in time & frequency**
  - ☞ remote comparison of clocks
  - ☞ statistics of non stationary processes
  - ☞ predictability of non stationary processes
- ✓ **some mathematical tools**
  - ☞ the time process x(t) and the frequency process y(t)
  - ☞ time domain statistics : the Allan deviation

summary (B)

- ✓ **applications of the Allan deviation**
  - ☞ measurement of the frequency stability
  - ☞ prediction of timescales
  - ☞ interpolation of timescales
- ✓ **uncertainty on time & frequency calibrations**
  - ☞ primary cesium standards and the definition of the SI second
  - ☞ uncertainty on timescale calibrations
  - ☞ uncertainty on frequency calibrations
- ✓ **prospects**
  - ☞ application of time & frequency tools to other fields of metrology

what is a clock ?

- ✓ **clock** = **frequency standard** + **counter + origin**

- ✓ the **frequency standard** produces a periodical output with a defined frequency (Hz) or period (s)
  - ☞ oscillation of a pendulum
  - ☞ oscillation of a quartz oscillator
  - ☞ rotation of the Earth (period 1 day)
  - ☞ cesium frequency standard (1 PPS, 5 MHz ouputs)

- ✓ the **counter** counts the periods and keeps track of the seconds, minutes, days, weeks, months, years elapsed since the initialisation of the clock

- ✓ the **origin** is the constant that links the time interval accumulated since initialisation to the epoch i.e. to the absolute time

what is a timescale ?

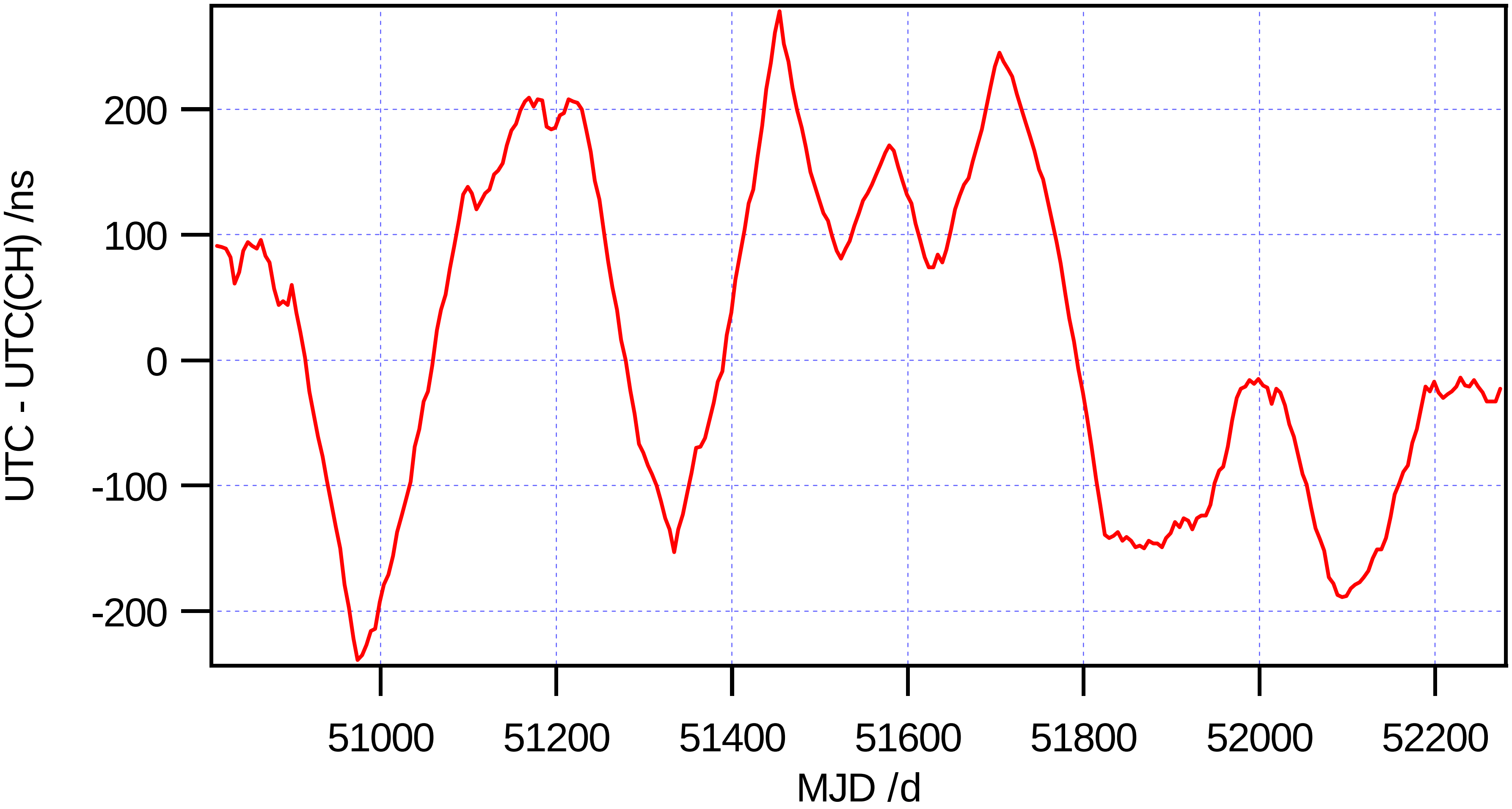
✓ epoch

- epoch = **absolute time** coordinate of an event
  i.e. the time in *year.month.day hh:mm:ss* at which
  a measurement is made or at which an event occurs

✓ timescale

- **output** of a clock
- **observable** realisation of time
- as produced by a clock or by an ensemble of clocks
- in the form of a **measurable** signal
- or an **observable** natural periodical process
- or of a **computable** « paper clock »
- a timescale is an imperfect realisation of the epoch
- only timescale **differences** can be measured

example of a timescale



epoch in MJD (Modified Julian Day)

international timescales (A)

✓ **UT1**

- **Universal Time**
- astronomical timescale generated by the observation of the rotation of the Earth (**1 period = 1d = 86400 s**)

✓ **TAI**

- **Temps Atomique International** computed by BIPM
- by collecting data from **200 atomic clocks** located in NMI timekeeping labs around the world
- steered in frequency to comply with the SI **s** unit by means of regular comparisons with **primary cesium standards** in NMI labs
- SI **s** unit defined as the duration of **9 192 631 770 periods** of the **cesium hyperfine resonance**

international timescales (B)

✓**UTC**

☞**Universal Time Coordinated** computed by BIPM

☞**UTC = TAI + *n* leap seconds**

☞now *n* = 32 s

✓ **leap seconds**

☞ UT1 **s** unit > SI **s** unit (rotation of Earth slowing down)

☞ UTC - UT1 > 0 with the difference growing

☞ the difference UTC - UT1 is monitored by IERS **International Earth Rotation Service** which decides when a **leap second** must be added to UTC so that the difference UTC - UT1 is never larger than **0.9 s**

☞ adding a leap second procuces a **61s minute** in UTC that makes UTC actually loose 1 s so that UT1 can catch up

☞ a leap second is necessary only every few years

☞ last leap second added 31.12.1998

Swiss timescales

- **TA(CH)**
  - **free running atomic timescale** generated at METAS
  - 7 commercial cesium frequency standards
  - « paper clock » time scale algorithm
  - clock data sent to BIPM for contribution to TAI
  - difference TAI-TA(CH) published monthly by BIPM
- **UTC(CH)**
  - **steered atomic timescale** generated at METAS
  - derived from TA(CH)
  - **steered in frequency** (every month) to track UTC
  - timescale data sent to BIPM
  - difference UTC-UTC(CH) published monthly by BIPM

METAS cesium clocks in N1U25

legal time and SI **s** unit in Switzerland

- ✓ **CET** (Central European Time)
  - ☞ **UTC is the basis for legal time** (equivalent to old GMT)
  - ☞ In Switzerland legal time is
  - ☞ CET = UTC + 1h (normal time)
  - ☞ CEST = UTC + 2h (summer time)

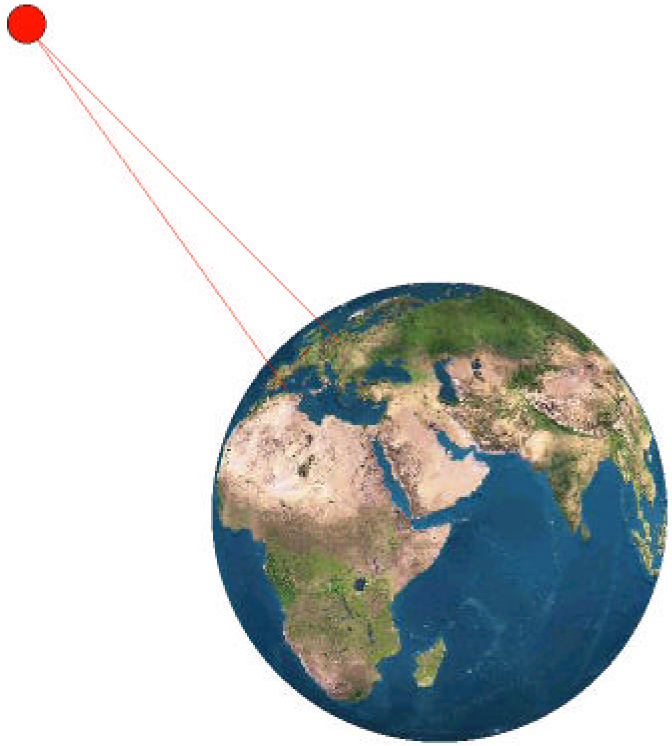
- ✓ **Key Comparisons**
  - ☞ Decision by CCTF in 2001 that the comparisons UTC-UTC(k) published in the **Annual Report of the Time & Frequency Section of BIPM** play the role of Key Comparison for the SI **s**

remote comparison of clocks

- ✓ remote comparison methods
  - ☞ **GPS common view**
  - ☞ TWSTFT (Two Way Satellite Time & Frequency Transfer)
  - ☞ carrier phase GPS (GeTT)
- ✓ GPS common view
  - ☞ precision : **1 ns** (1 day average, <1000 km baseline)
  - ☞ accuracy : **10 ns** (depends on **definition of reference plane)**
- ✓ example of metrological problem in time & frequency
  - ☞ **how to calibrate the reference plane ?**

GPS common view (A)

Earth as seen by GPS BIIA-27 (PRN 30)
20 '148 km above 13°59 'N 48°49 'E



✓GPS satellites
- ☞ 20'000 km altitude
- ☞ atomic clock on board

✓principle of GPS **common view**
- ☞ 2 stations track the **same satellite**
- ☞ **simultaneously** (15 minute tracks)
- ☞ common view schedule (BIPM)
- ☞ **differential measurement** eliminates common mode GPS clock error
- ☞ M1 = REF(LOC1) - REF(GPS)
- ☞ M2 = REF(LOC2) - REF(GPS)
- ☞ M2 - M1 = REF(LOC2) - REF(LOC1)

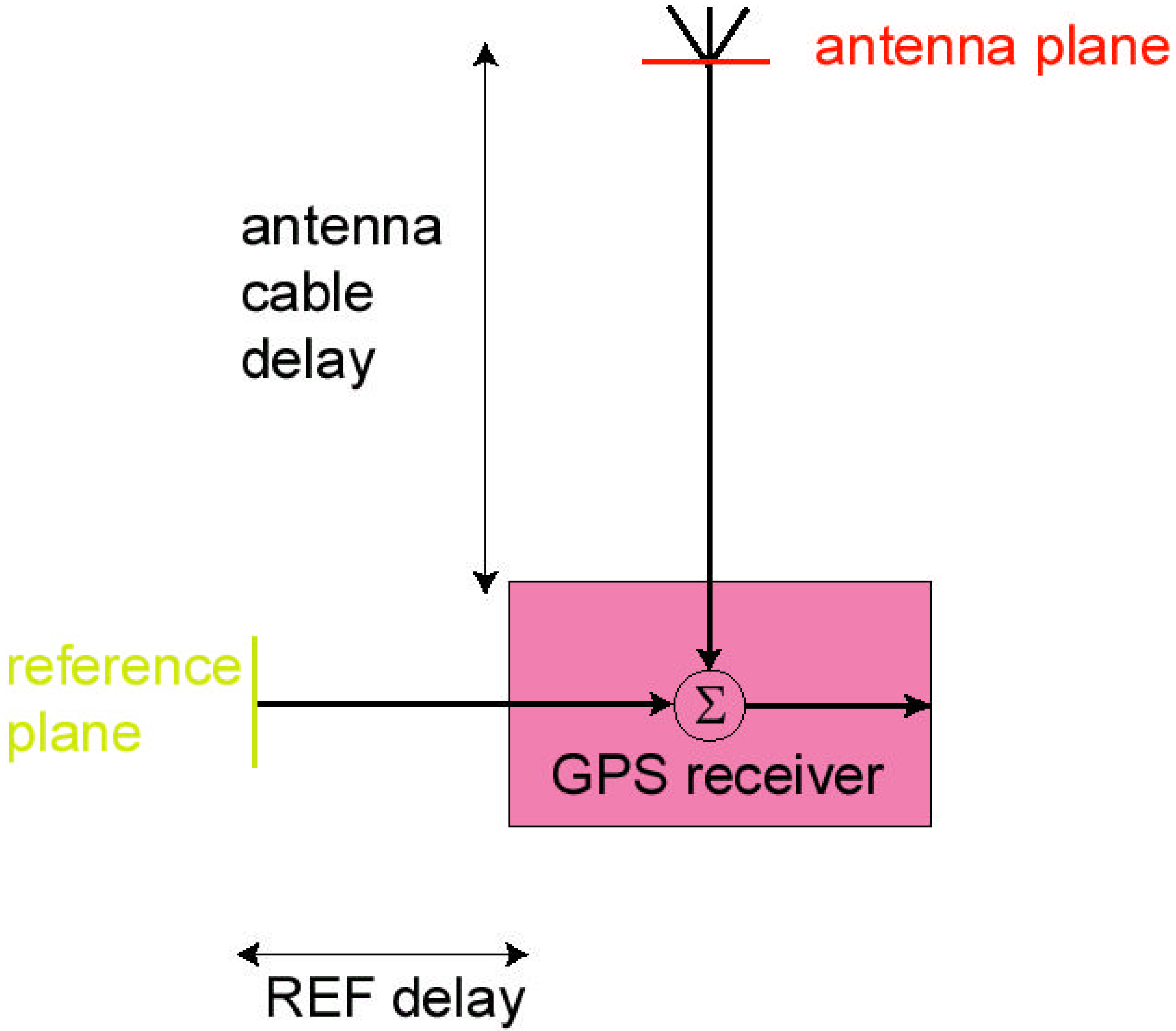
GPS common view (B)

- ✓ calibration of reference plane
  - ☞ accurate antenna coordinates must be known
  - ☞ receiver computes GPS-REF as seen by receiver
  - ☞ **accurate time transfer** is possible if
  - ☞ GPS receiver time is translated to **antenna plane**
  - ☞ REF receiver time is translated to **reference plane**
  - ☞ **antenna cable delay** and **reference delay** can be measured
  - ☞ **internal delay** can be calibrated only by comparison with **BIPM reference receiver** (calibration trip)

$$\left[GPS_{\text{antenna plane}} - REF_{\text{reference plane}}\right] = \left[GPS_{\text{receiver}} - REF_{\text{receiver}}\right] - DLY_{\text{total}}$$

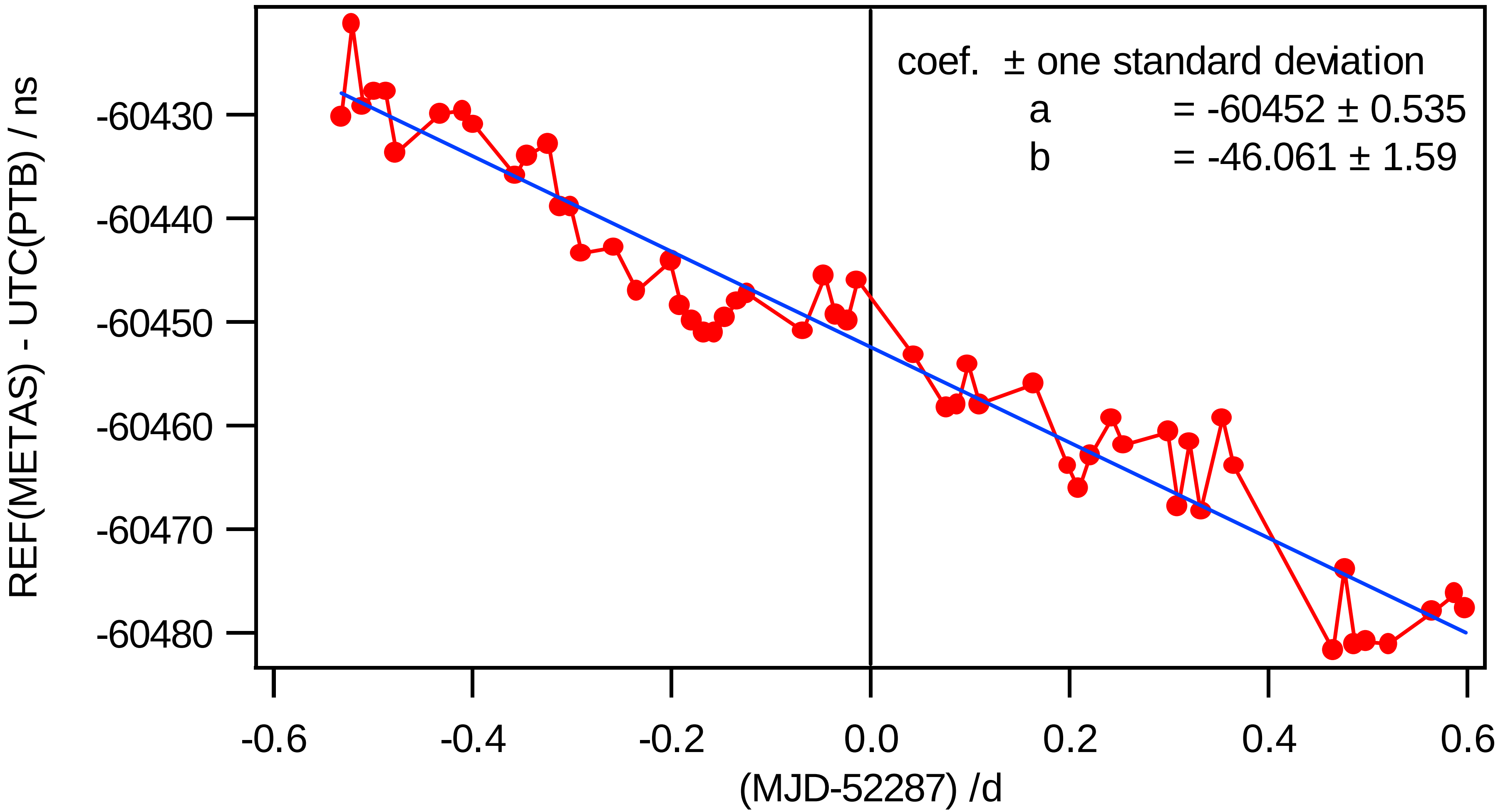
$$DLY_{\text{total}} = DLY_{\text{antenna cable}} + DLY_{\text{internal}} - DLY_{\text{reference}}$$

GPS common view (C)



GPS common view (D)

✓example of common view comparison between METAS and PTB



non-stationary statistics of timescales

✓ **frequency** of a clock or timescale
- ☞ the frequency random process is **not stationary**
- ☞ statistical properties are a function of time
- ☞ the **mean value** averaged over time diverges
- ☞ the **standard deviation** averaged over time diverges

✓ **time** of a clock or timescale
- ☞ time process is the **integral** of the frequency process
- ☞ if frequency is not stationary then time is even worse…
- ☞ for example a **white frequency noise process**…
- ☞ becomes a **random walk time process** (brownian motion)

✓ special statistical tools are needed (**Allan deviation**) for the characterisation and prediction of timescale random processes

prediction of timescales

- ✓ **Circular T**
  - ☞ TAI-TA(K) and UTC-UTC(K)
  - ☞ are published **monthly** by BIPM in **Circular T**
  - ☞ published on the 15th of a month covering previous month
  - ☞ **computational delay** between 15d and 30d

- ✓ **computation of UTC(CH) and TA(CH)**
  - ☞ UTC(CH) and TA(CH) are computed every day at 04:00 UTC for 00:00 UTC of the previous day
  - ☞ **computational delay** between 28h and 48h

- ✓ **clock calibration for customers**
  - ☞ **real time** calibration performed against **predicted timescale**
  - ☞ **post-processed** calibration performed against true timescale after publication of timescale data

time and frequency processes

✔ **time process x(t)**

☞ usual unit : ns

$$x(t) = timescale_2(t) - timescale_1(t)$$

✔ **frequency process y(t)**

☞ usual unit Hz/Hz (relative frequency) or ns/d (clock rate)

$$y(t) = \frac{dx(t)}{dt}$$

Allan deviation : definition (A)

- ✓ the Allan deviation is defined as the **rms change of the average frequency y(t,**t**)** between two successive measurements for an averaging interval tau
- ✓ it was introduced as a **measurement of frequency stability** to deal with the non stationarity of the frequency in frequency standards (true standard deviation does not exist)

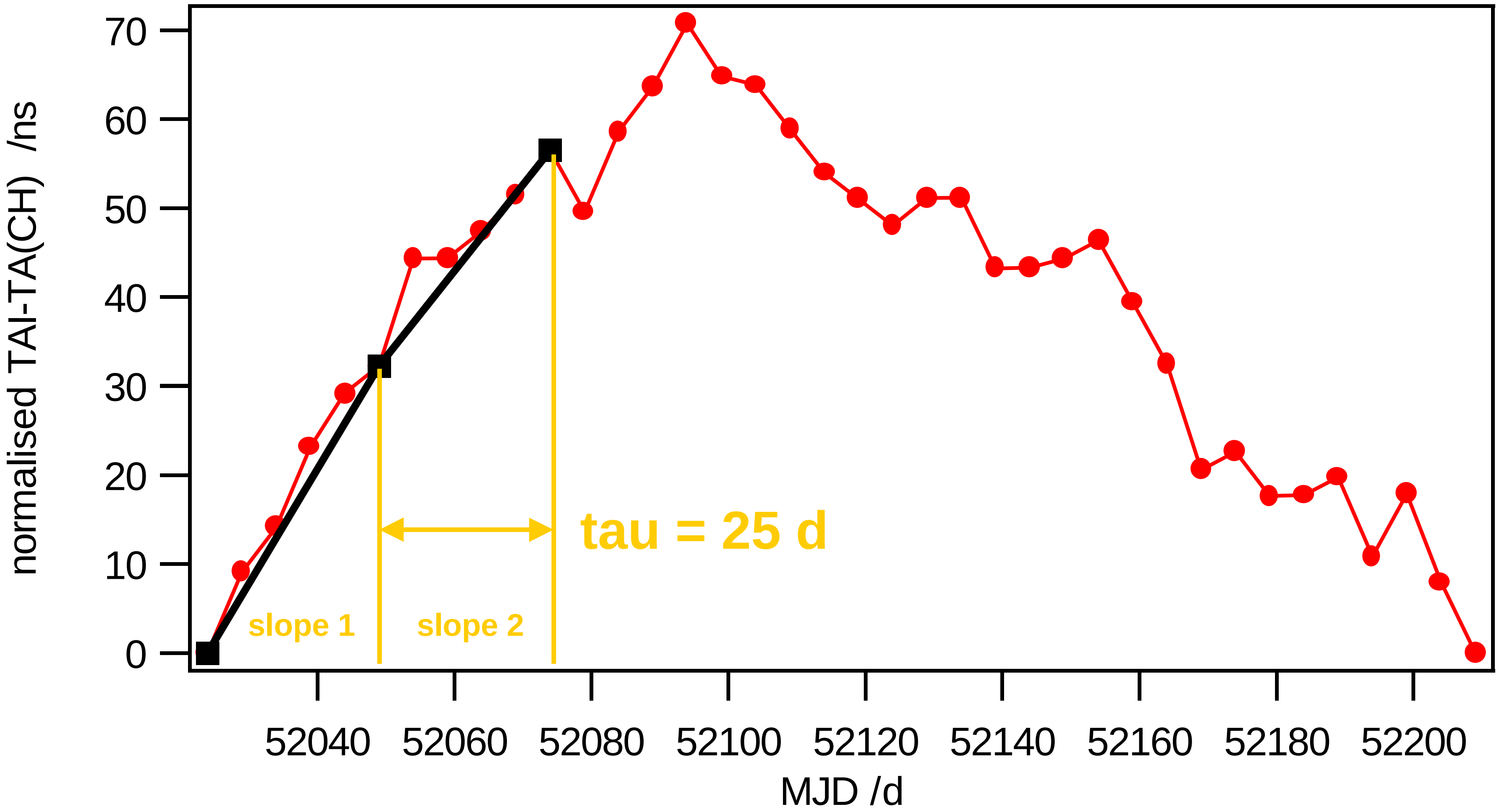
$$s_y(t) = \frac{1}{\sqrt{2}} rms\{y(t,t) - y(t-t,t)\}$$

Allan deviation : definition (B)

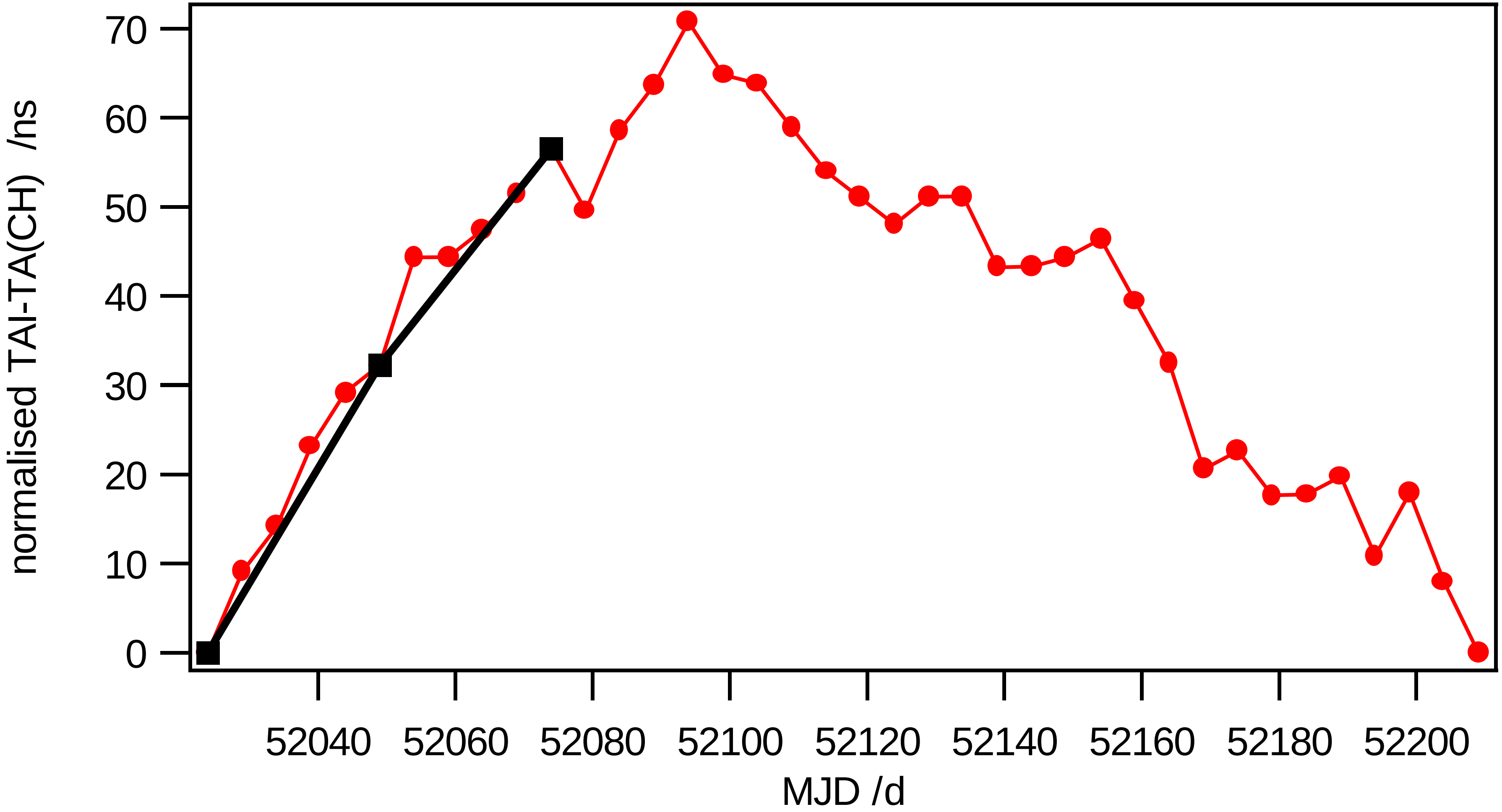
- ✓ **the average frequency y(t,**τ**)** over an averaging interval tau is identical to the **slope of x(t)** over an interval tau
- ✓ therefore the **Allan deviation** can be defined as the **rms change of the slope of x(t)** between two successive intervals tau
- ✓ the Allan deviation process is the **second order increment of x(t)**
- ✓ this process is stationary and has a defined standard deviation... … the Allan deviation

$$\sigma_y(\tau) = \frac{1}{\sqrt{2}} rms\left\{\left[\frac{x(t) - x(t-\tau)}{\tau}\right] - \left[\frac{x(t-\tau) - x(t-2\tau)}{\tau}\right]\right\}$$

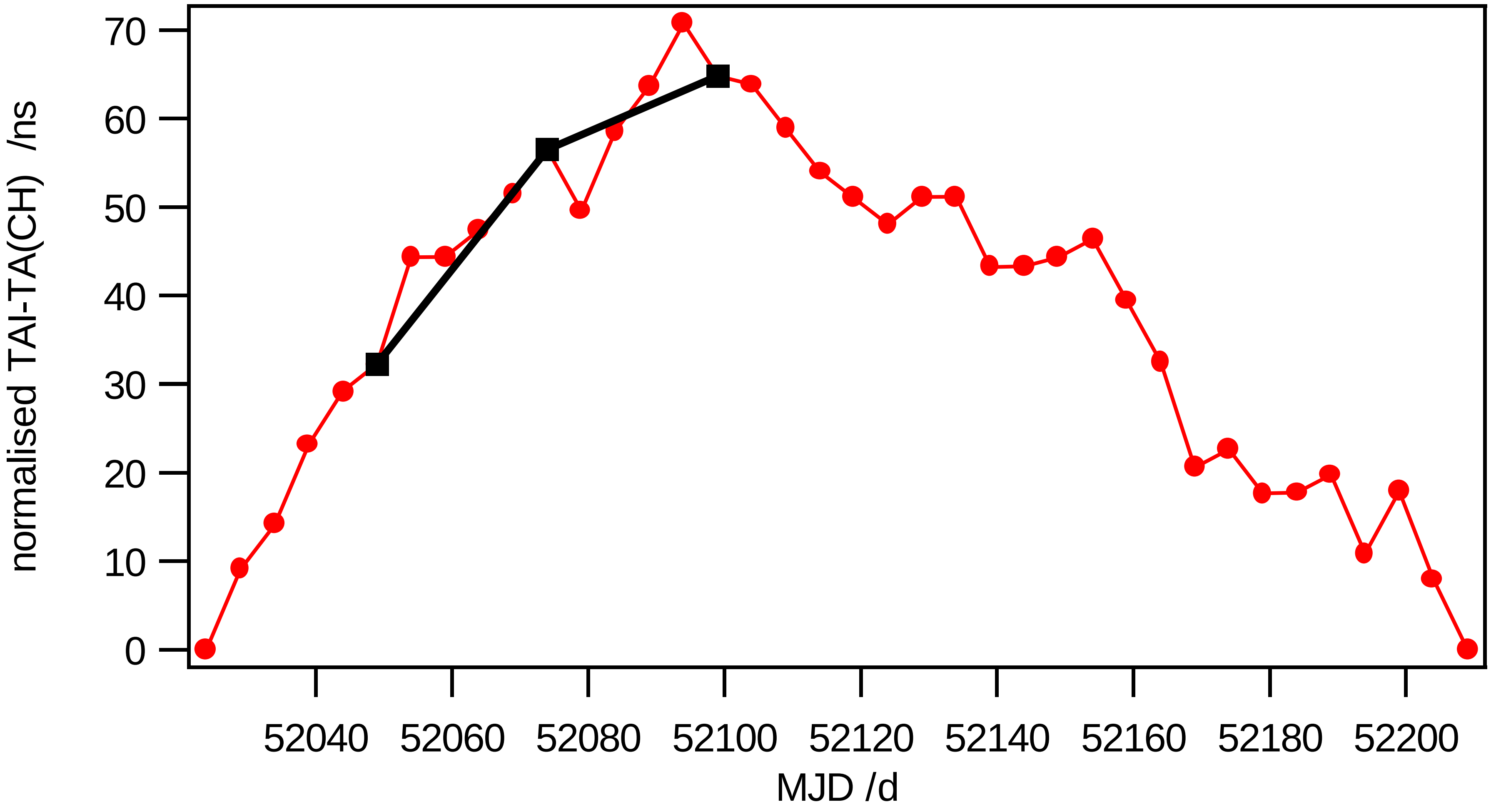
Allan deviation : example 1



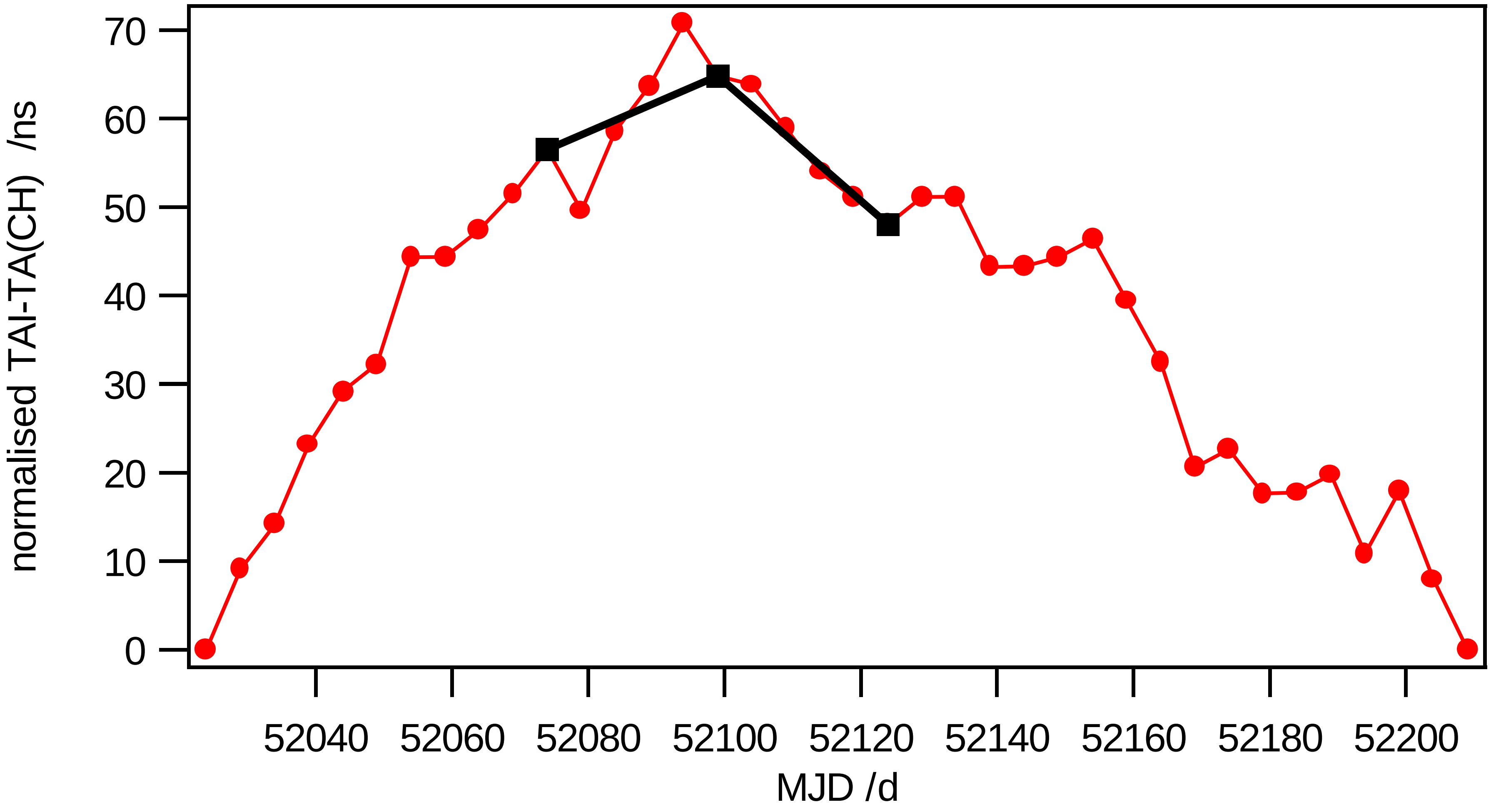
Allan deviation : example 1



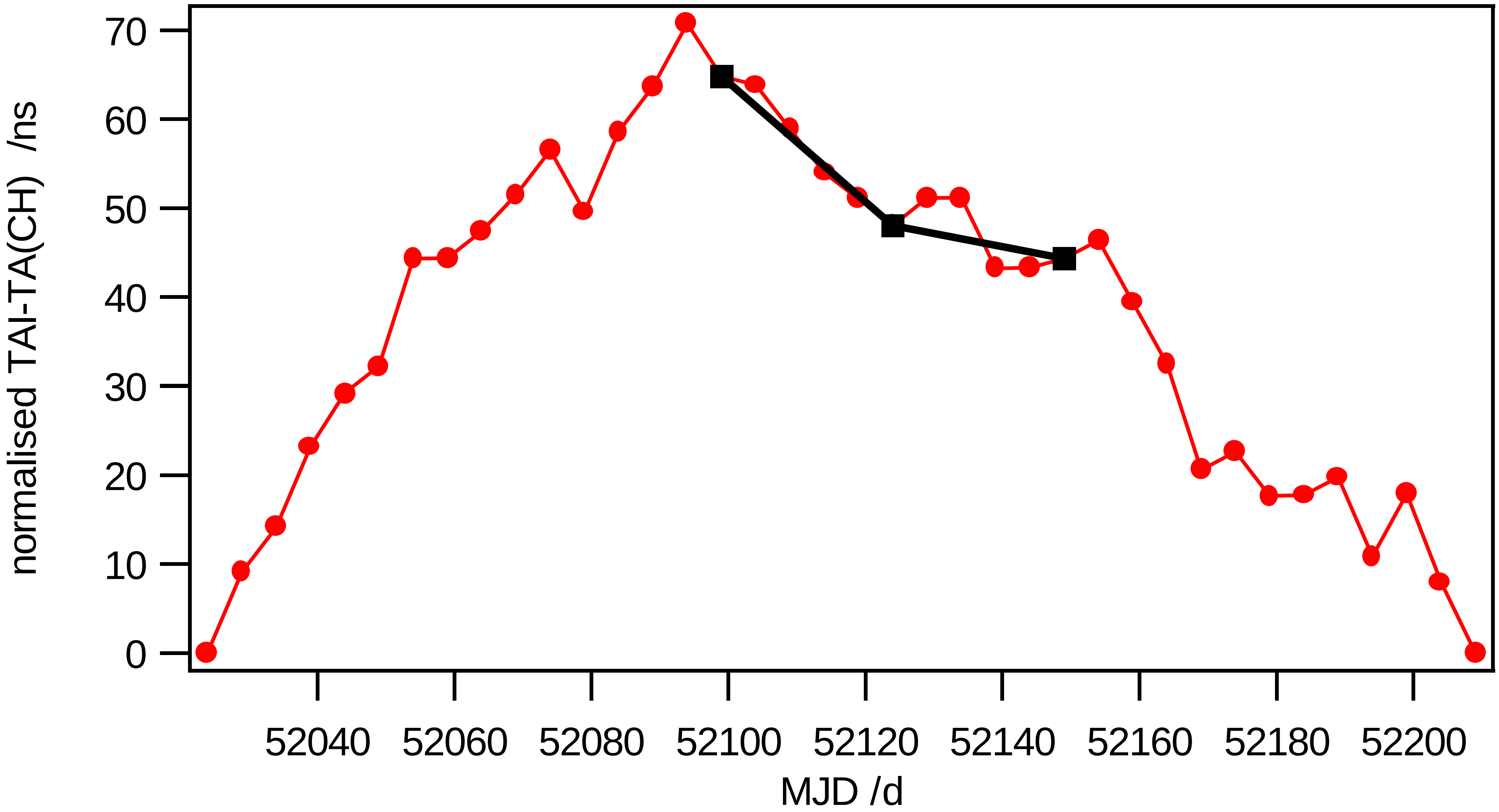
Allan deviation : example 1



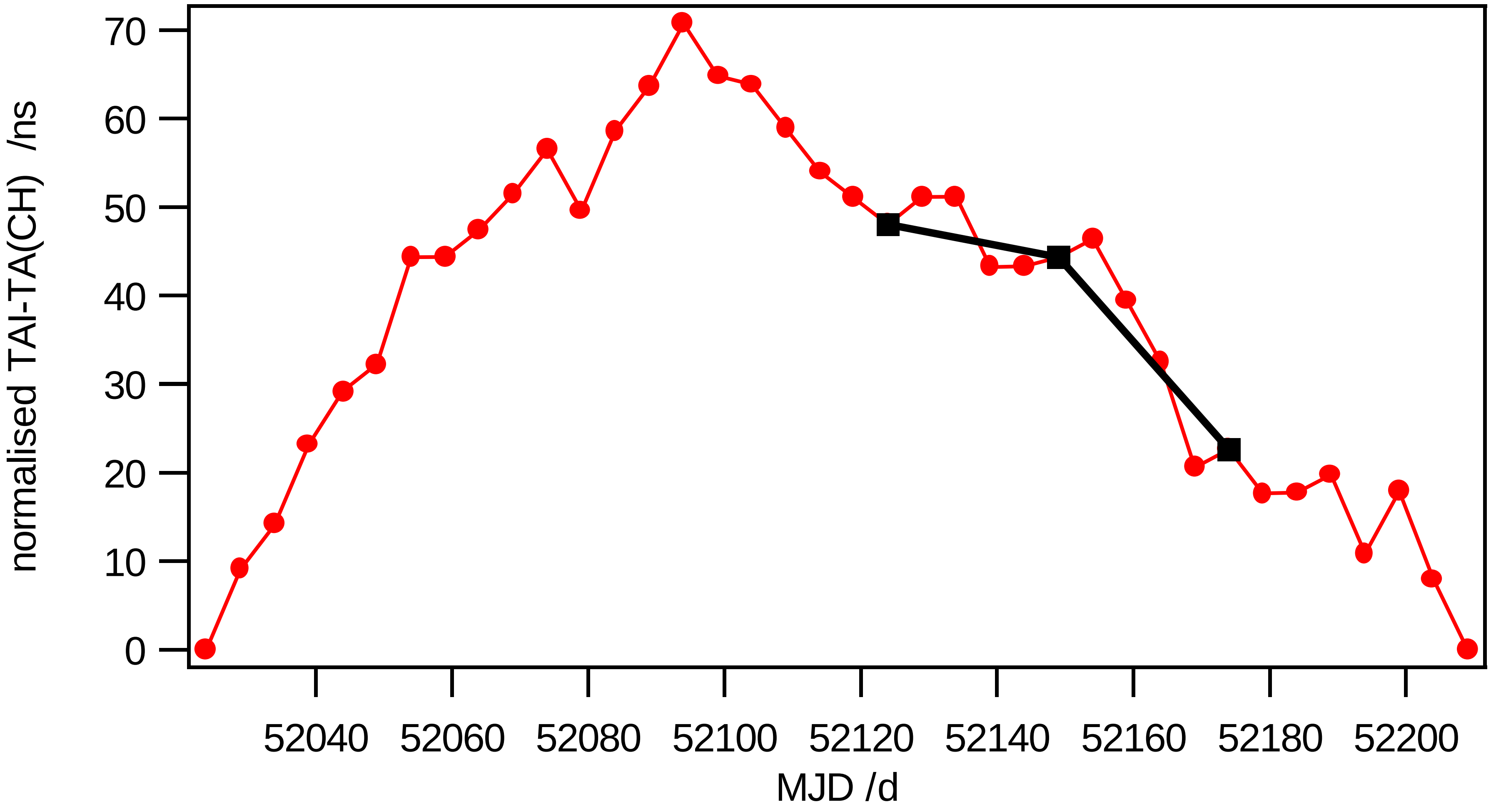
Allan deviation : example 1



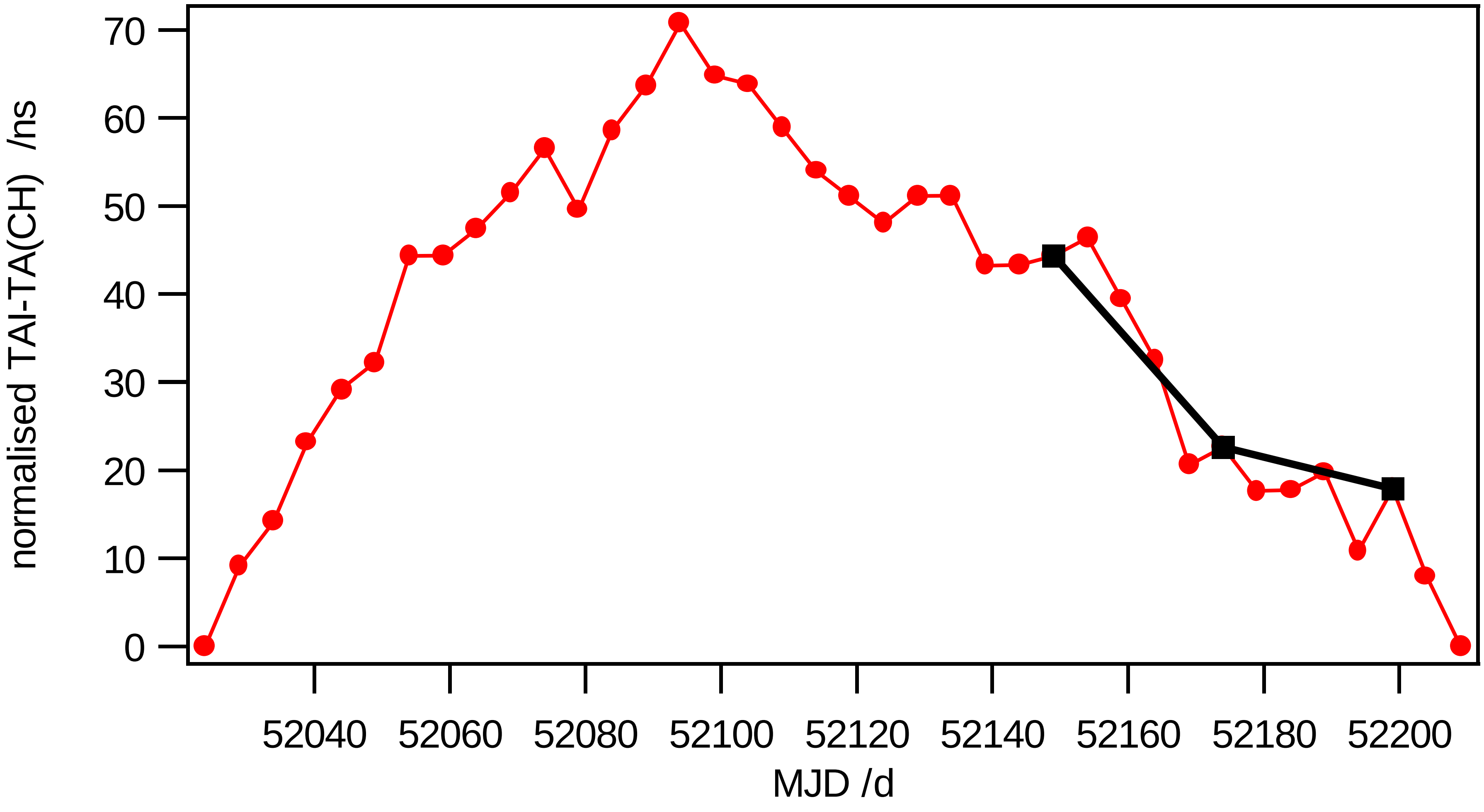
Allan deviation : example 1



Allan deviation : example 1



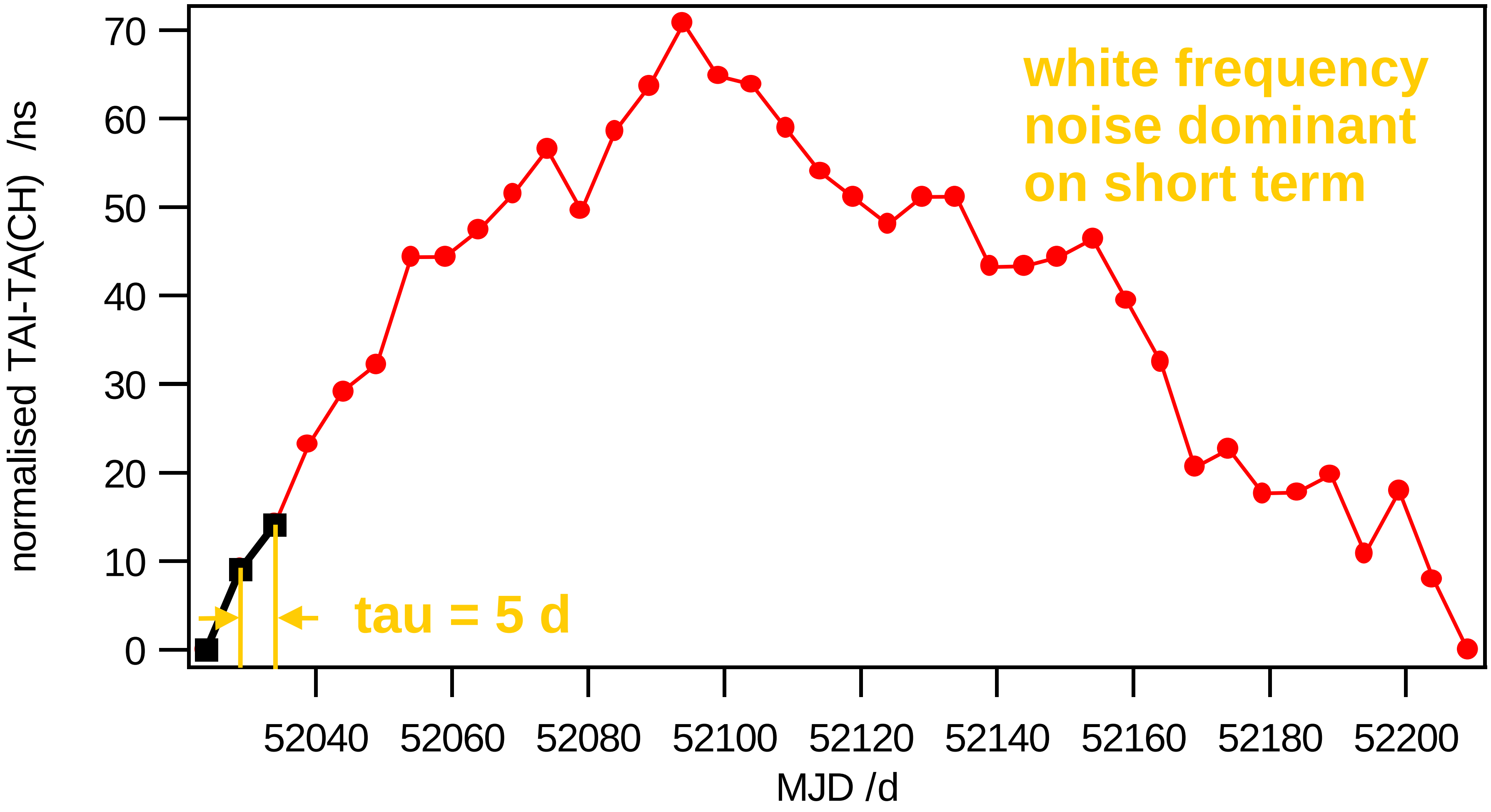
Allan deviation : example 1



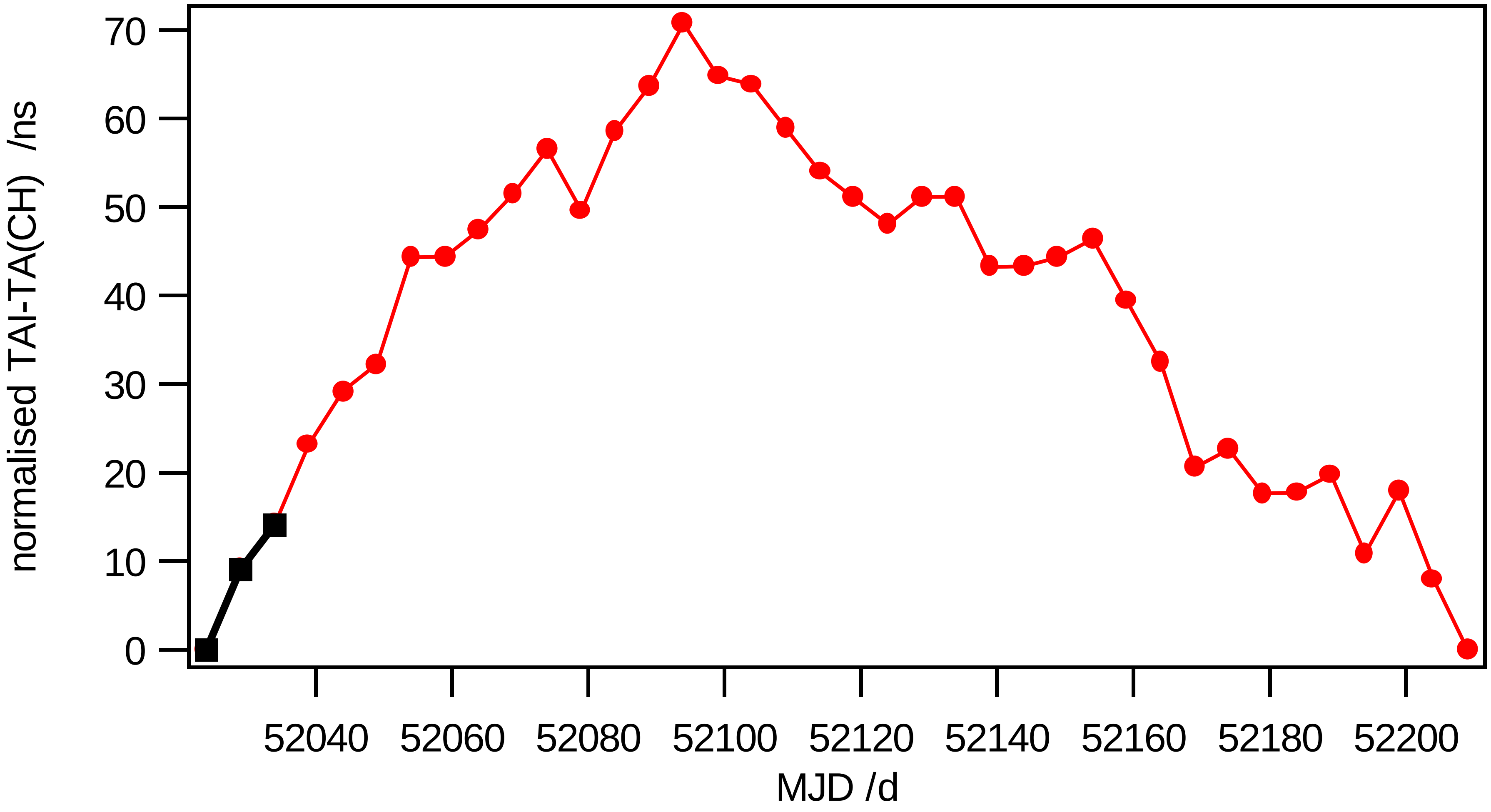
Allan deviation : properties (A)

- ✓ the **Allan deviation** is a time domain statistic that changes as a **function** of the averaging interval **tau**
- ✓ like a **fractal :** statistical properties of the timescale process are a function of the length of the **ruler** used to measure it
- ✓ example : timescale based on cesium clocks
  - ☞ **white frequency noise** (frequency stationary) dominates over short term averaging intervals
  - ☞ **flicker of frequency** (frequency non stationary) dominates over long term averaging intervals

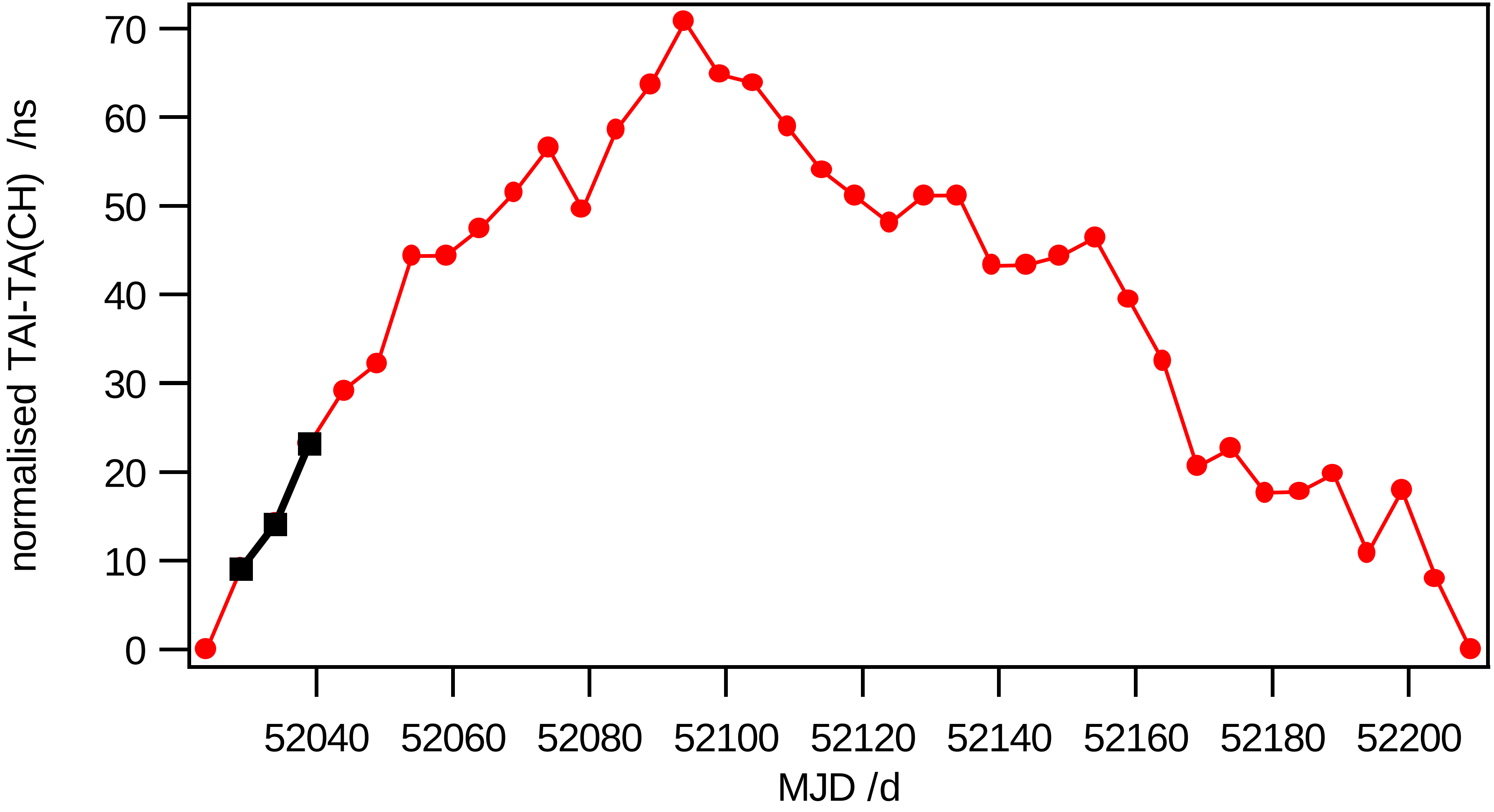
Allan deviation : example 2



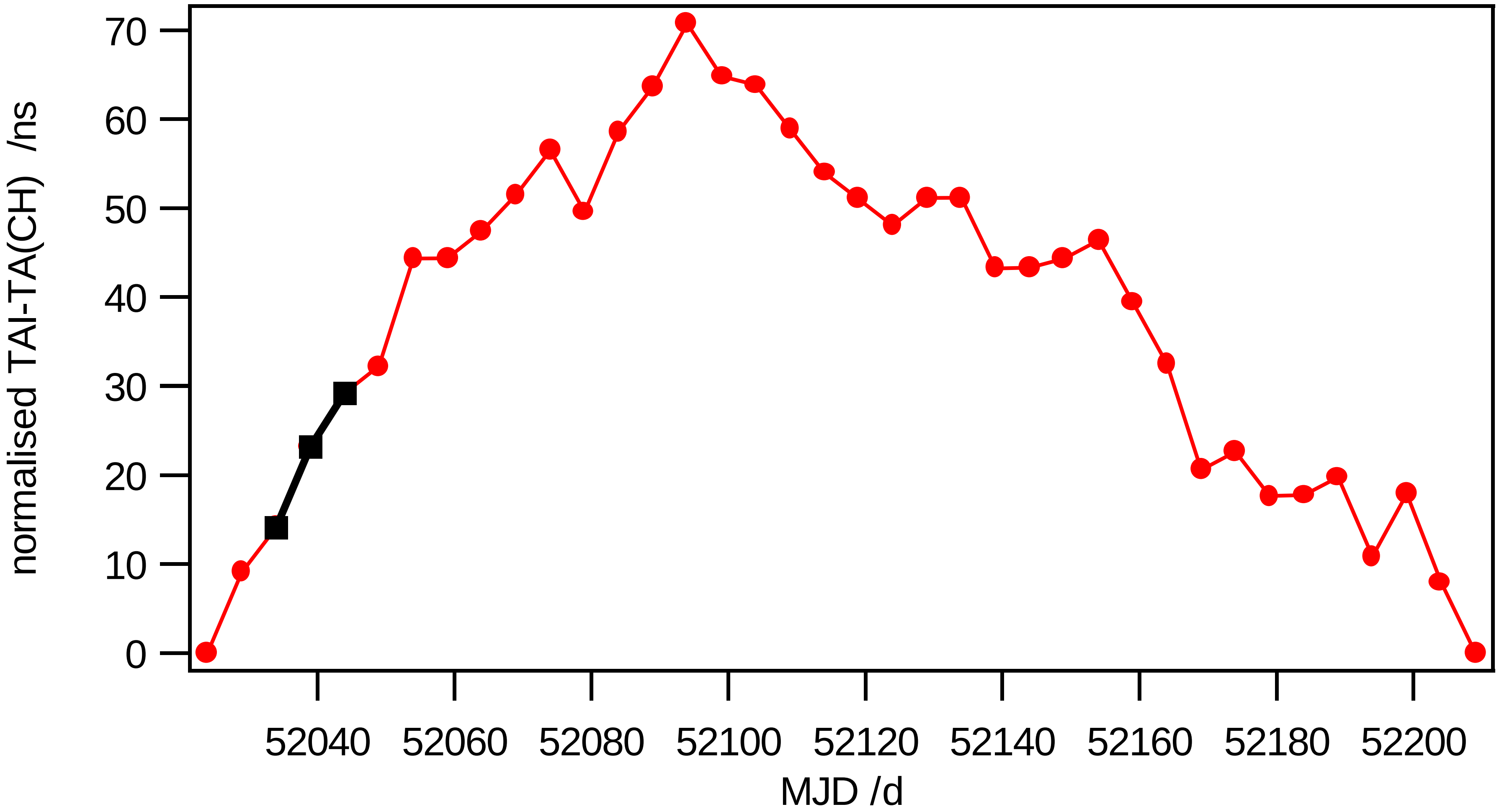
Allan deviation : example 2



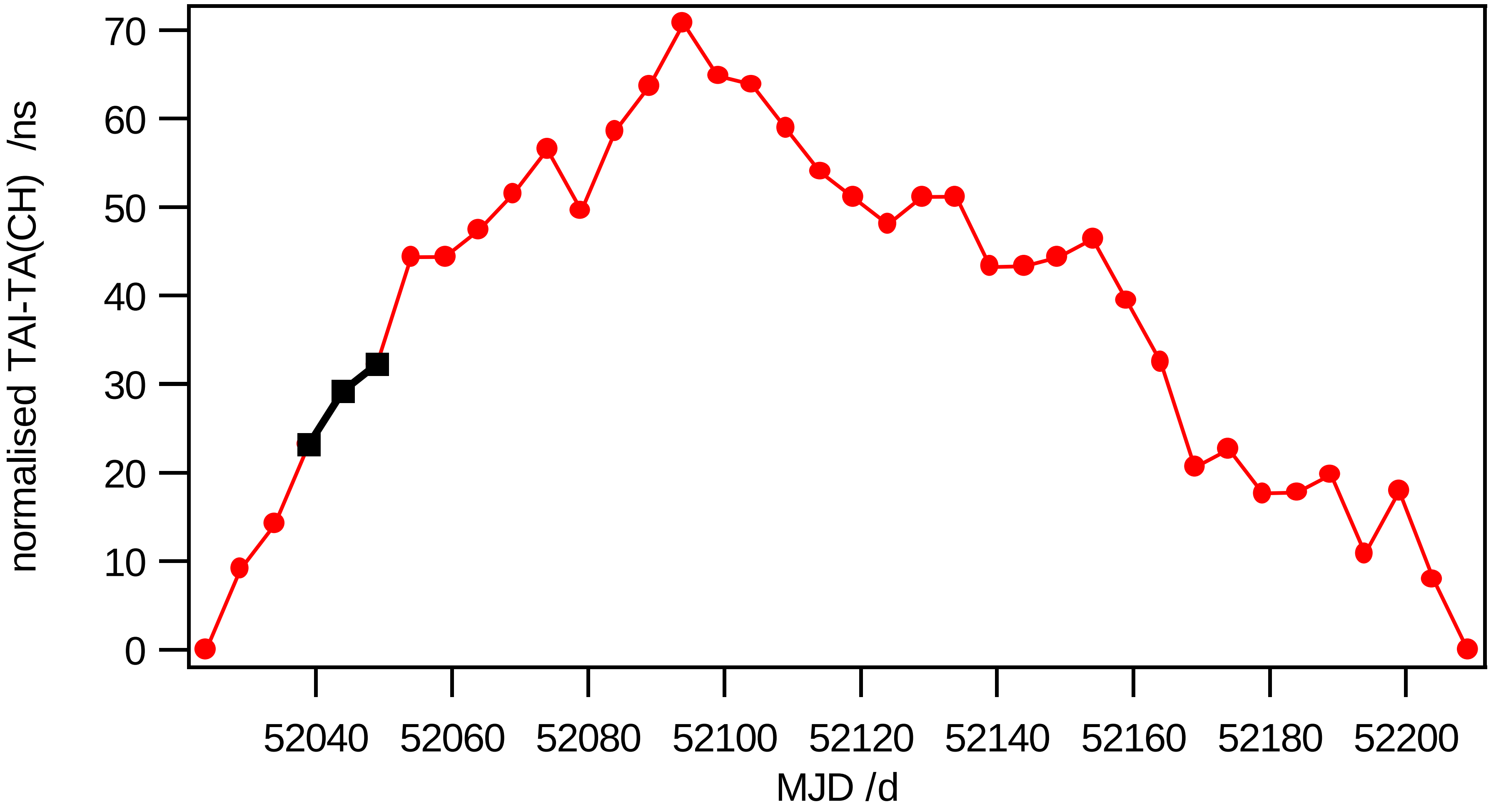
Allan deviation : example 2



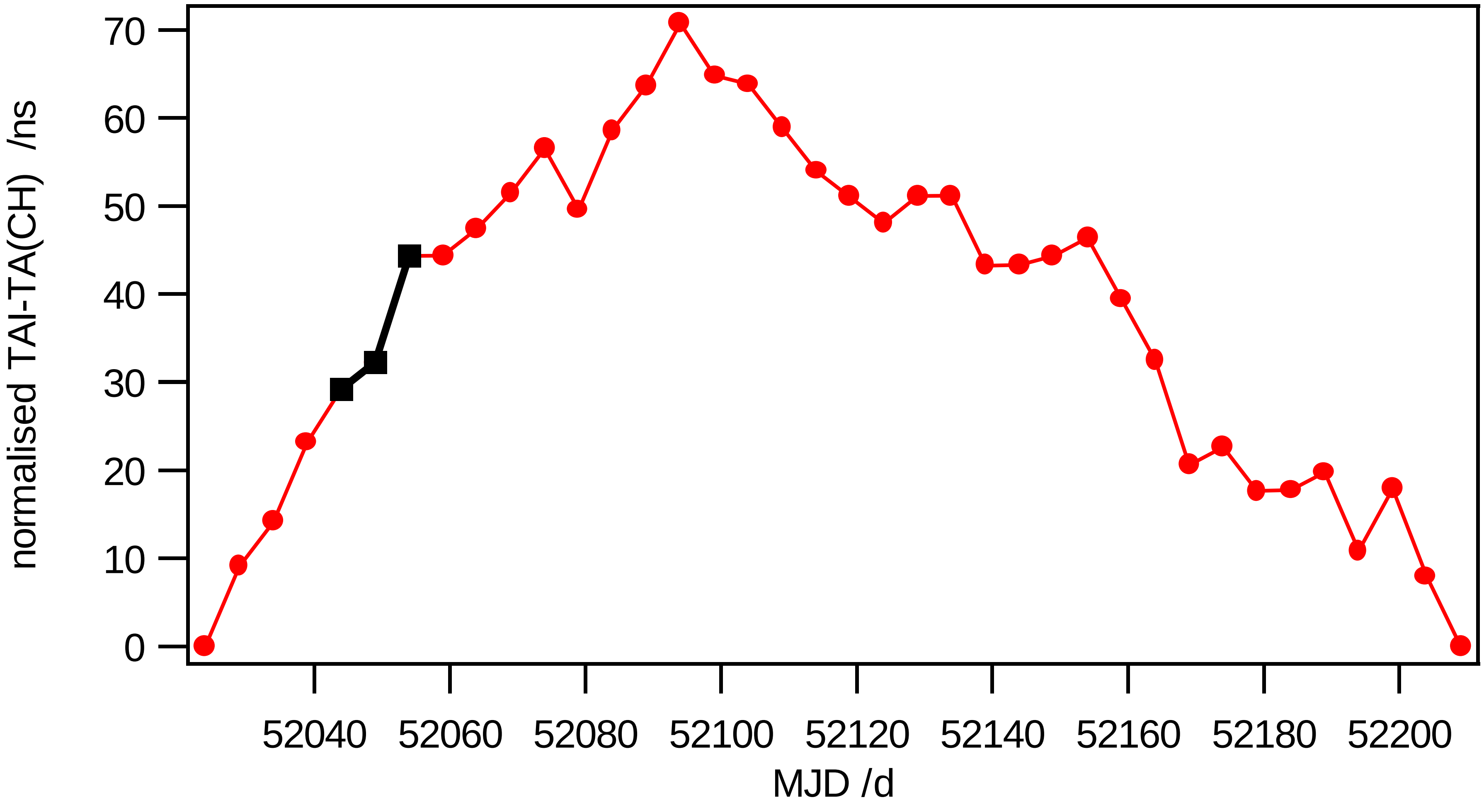
Allan deviation : example 2



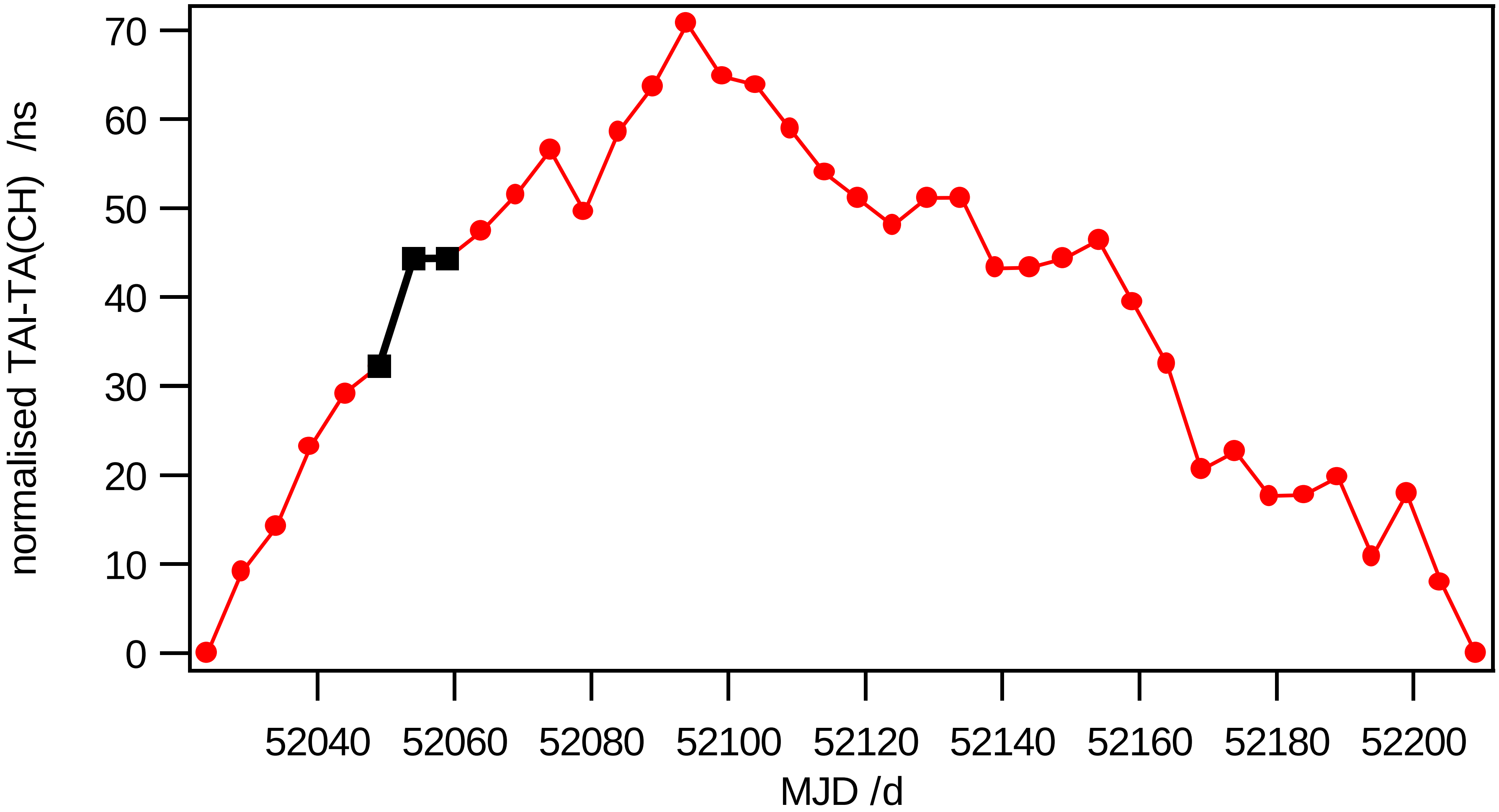
Allan deviation : example 2



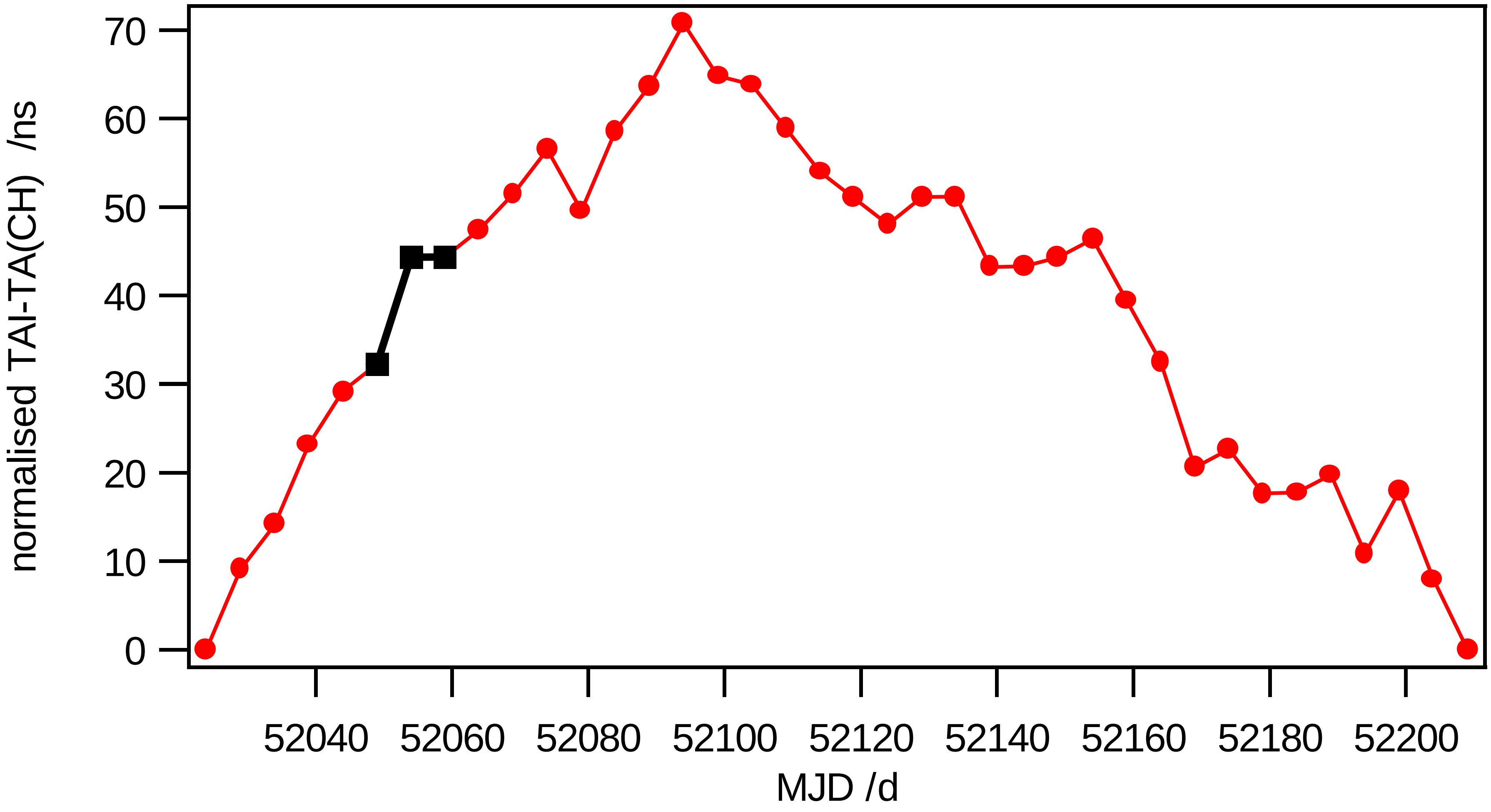
Allan deviation : example 2



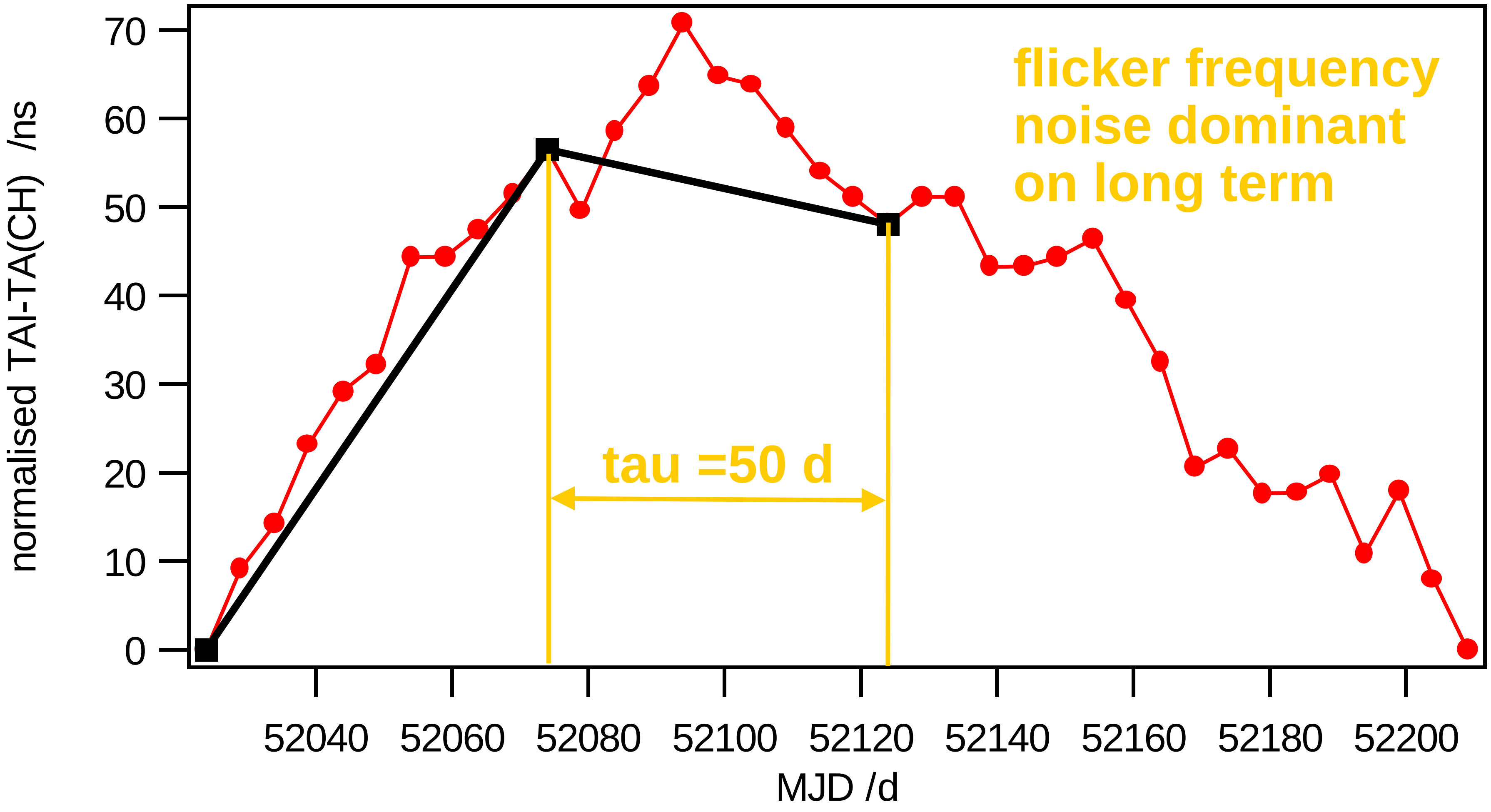
Allan deviation : example 2



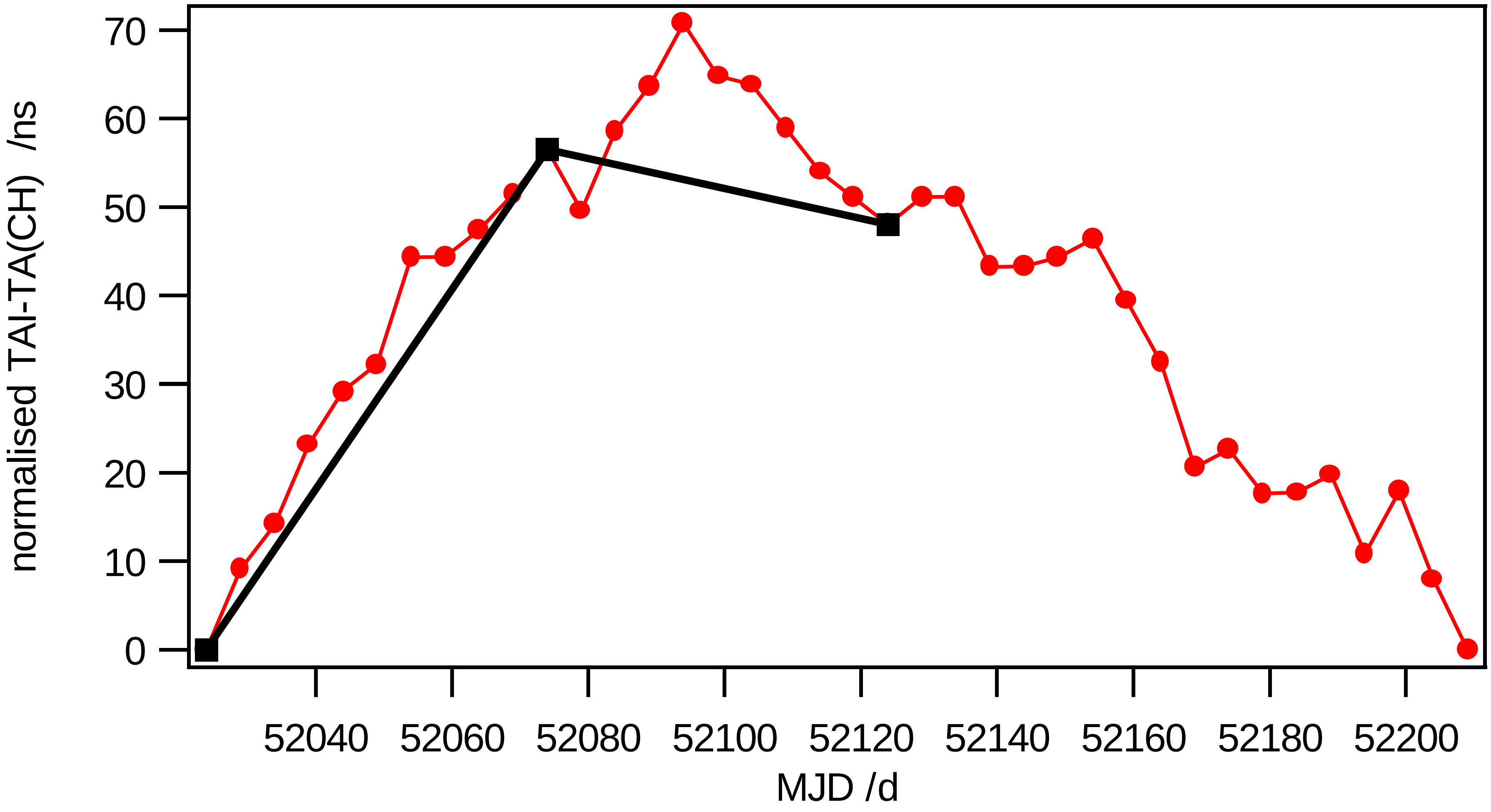
Allan deviation : example 2



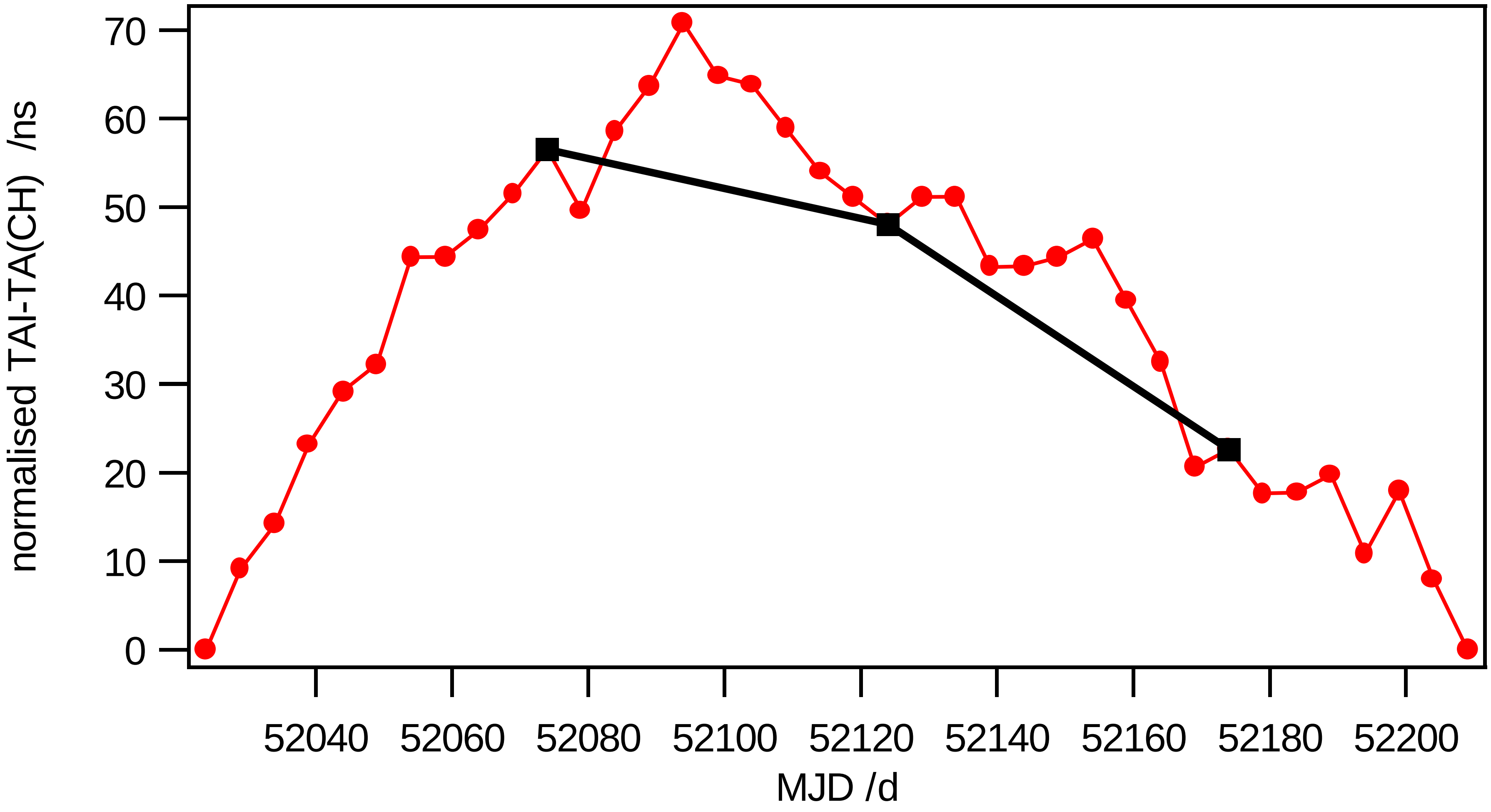
Allan deviation : example 3



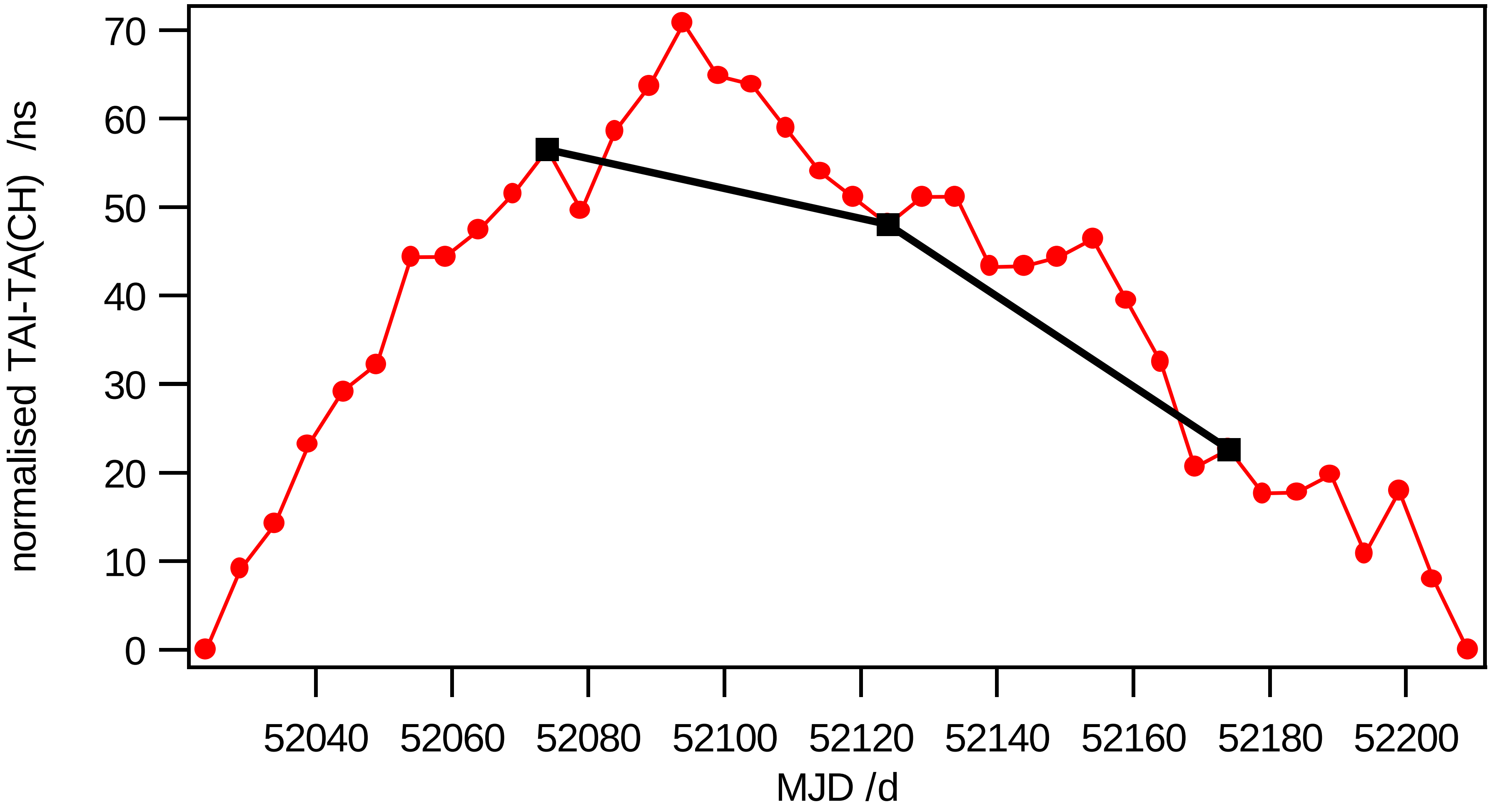
Allan deviation : example 3



Allan deviation : example 3

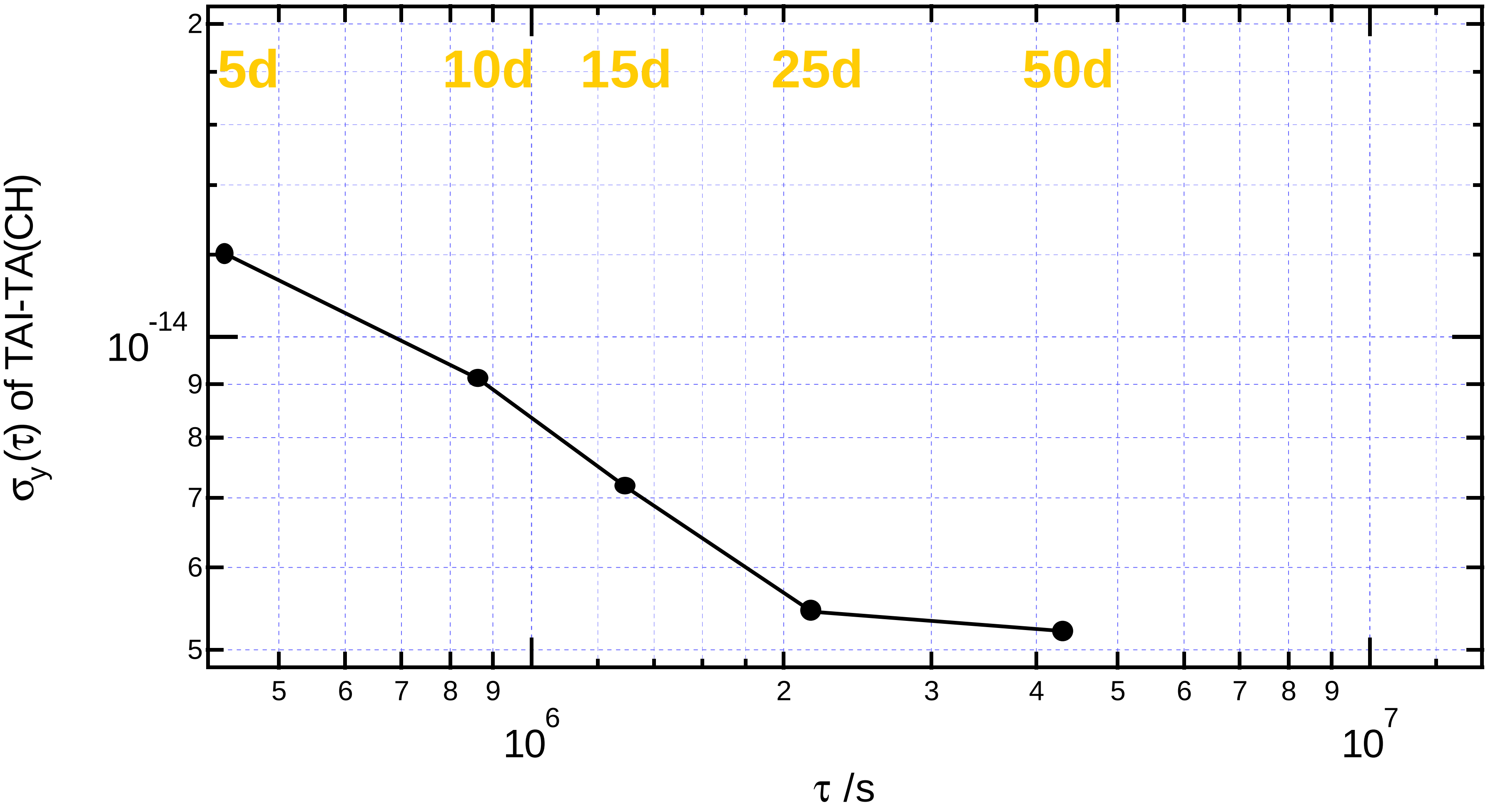


Allan deviation : example 3



Allan deviation : example 4

✓ **Allan deviation** of **TAI-TA(CH)**



Allan deviation : properties (B)

- ✓ **white frequency** noise process
  - ☞ Allan deviation is characterised by a **slope -1/2** versus tau in LOG-LOG scales
  - ☞ the Allan deviation improves with averaging time
  - ☞ a typical stationary process
  - ☞ this is similar to the standard deviation that improves with the averaging time when noise is stationary
- ✓ **flicker of frequency** noise process
  - ☞ the Allan deviation is independent of tau **(slope 0)**
  - ☞ a typical non stationary process

linear prediction of a timescale

✓ **definition of the predictor**

- ☞ rms frequency change is Allan deviation
- ☞ average frequency change is zero
- ☞ on average frequency will stay the same over next interval
- ☞ predicted = present + interval x slope + error

$$x(t+t)=x(t)+t\times y(t,t)+e(t+t)$$

✓ **error on the prediction**

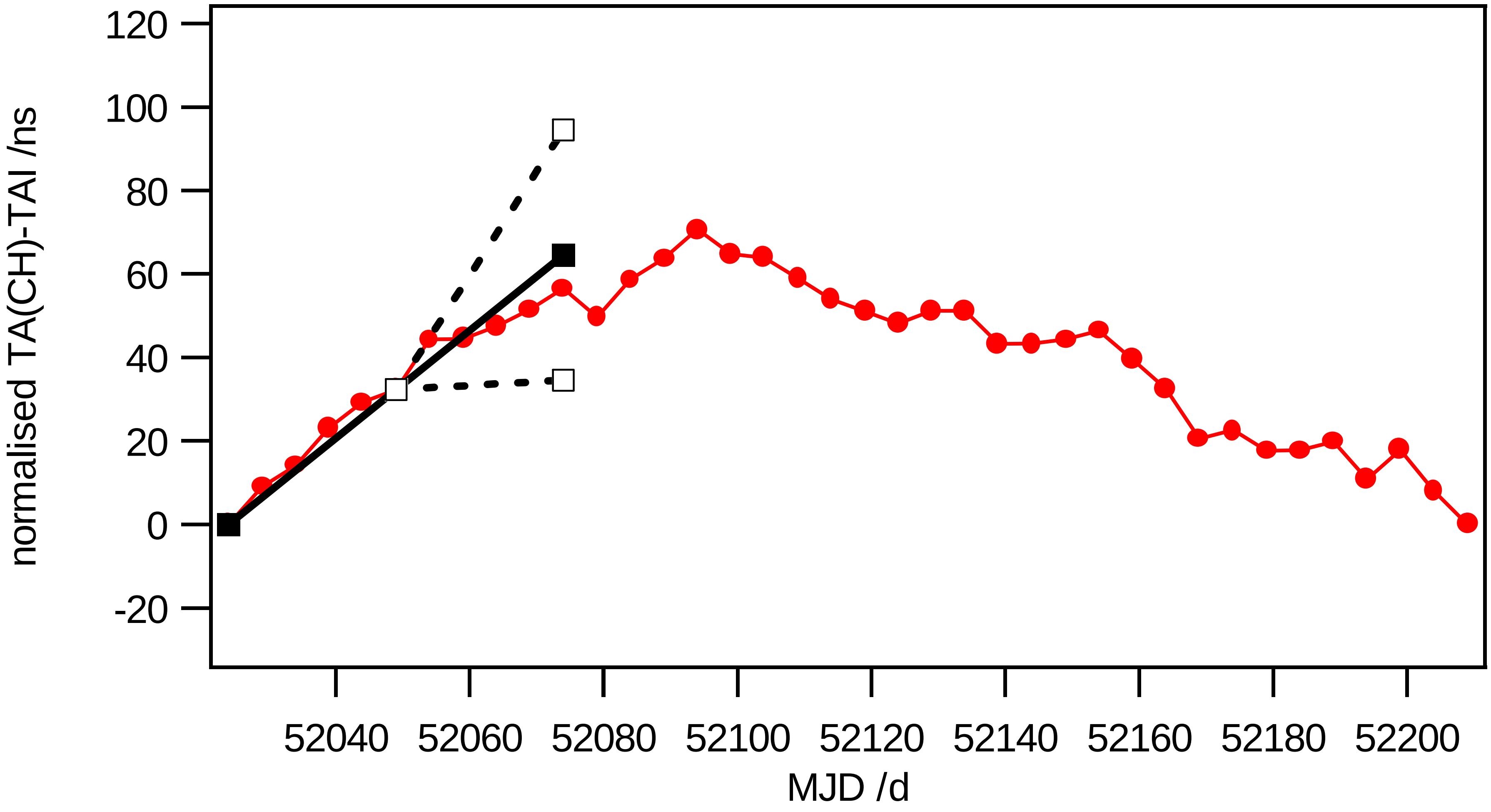
- ☞ **rms error** on the prediction proportional to the **Allan deviation**

$$rms\{e(t+t)\}=\sqrt{2}\times t\times s_{y}(t)$$

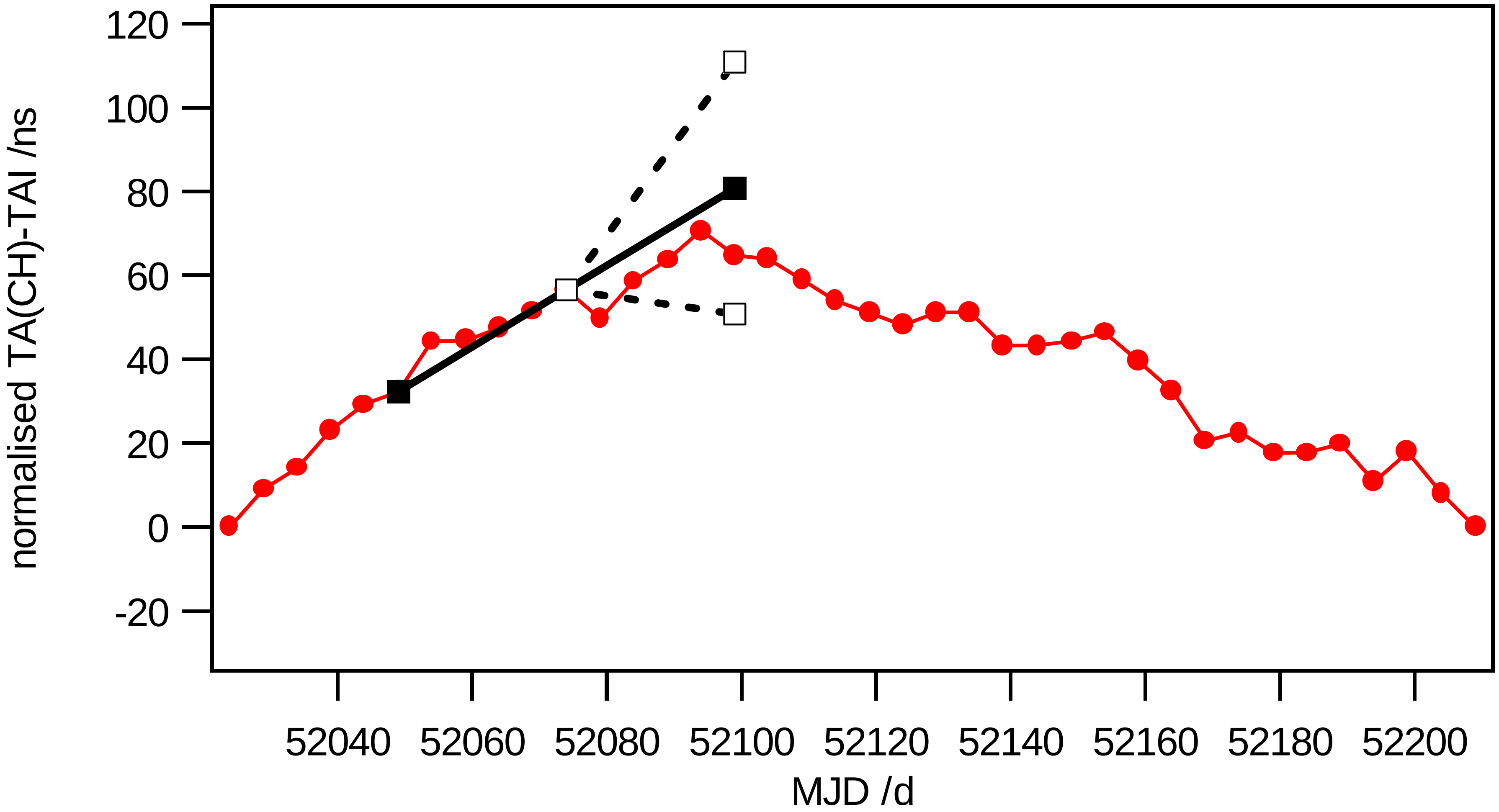
linear prediction : example

- ✓ prediction of TAI-TA(CH)
- ✓ prediction interval tau = 25 d
- ✓ 95% probability interval = 2 x rms prediction error
- ✓ Allan deviation is 5 x 10^{-15} over 25 days
- ✓ 95% probability interval of prediction uncertainty is **± 30 ns** over 25 days

linear prediction : example

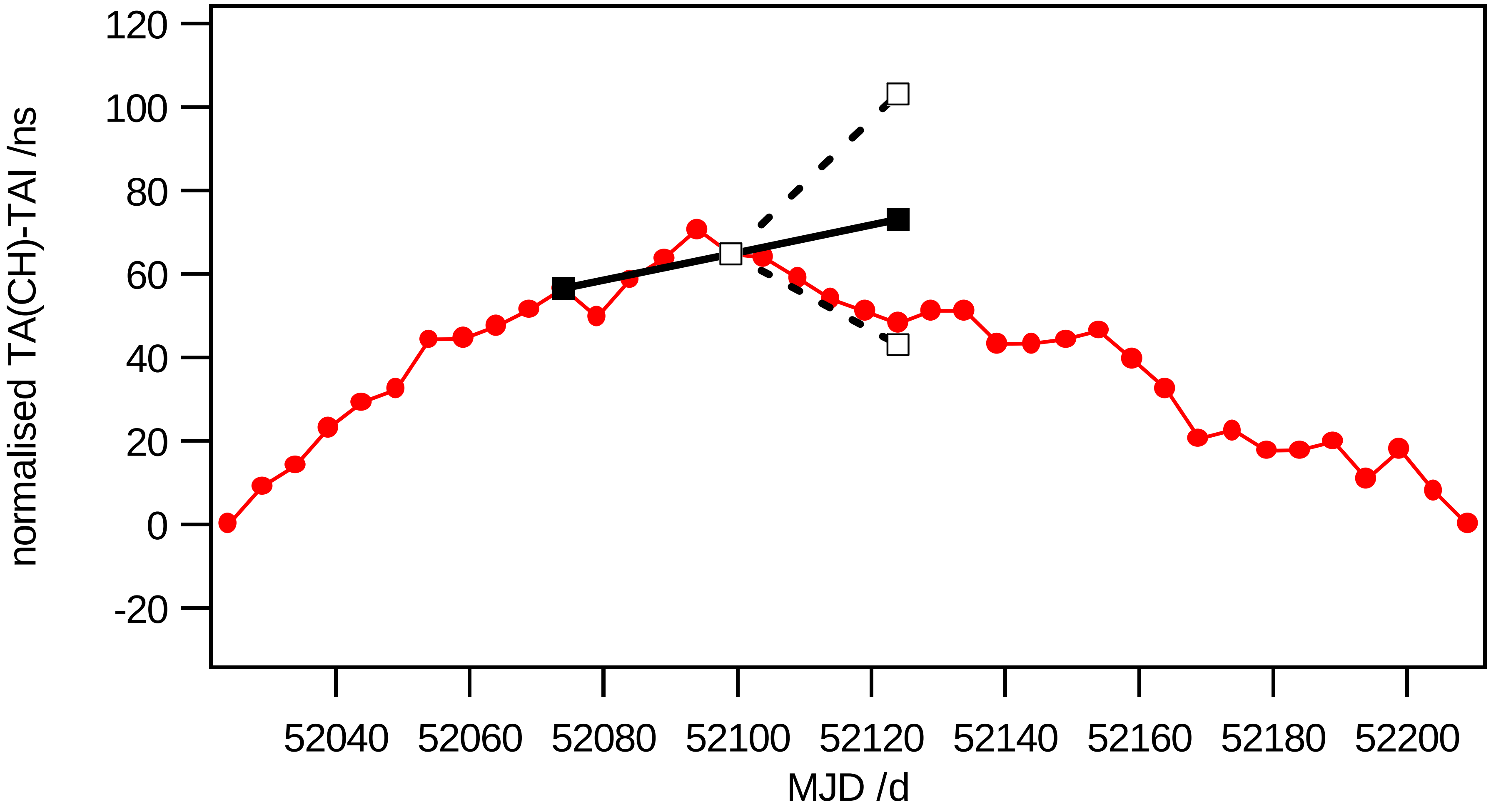


linear prediction : example

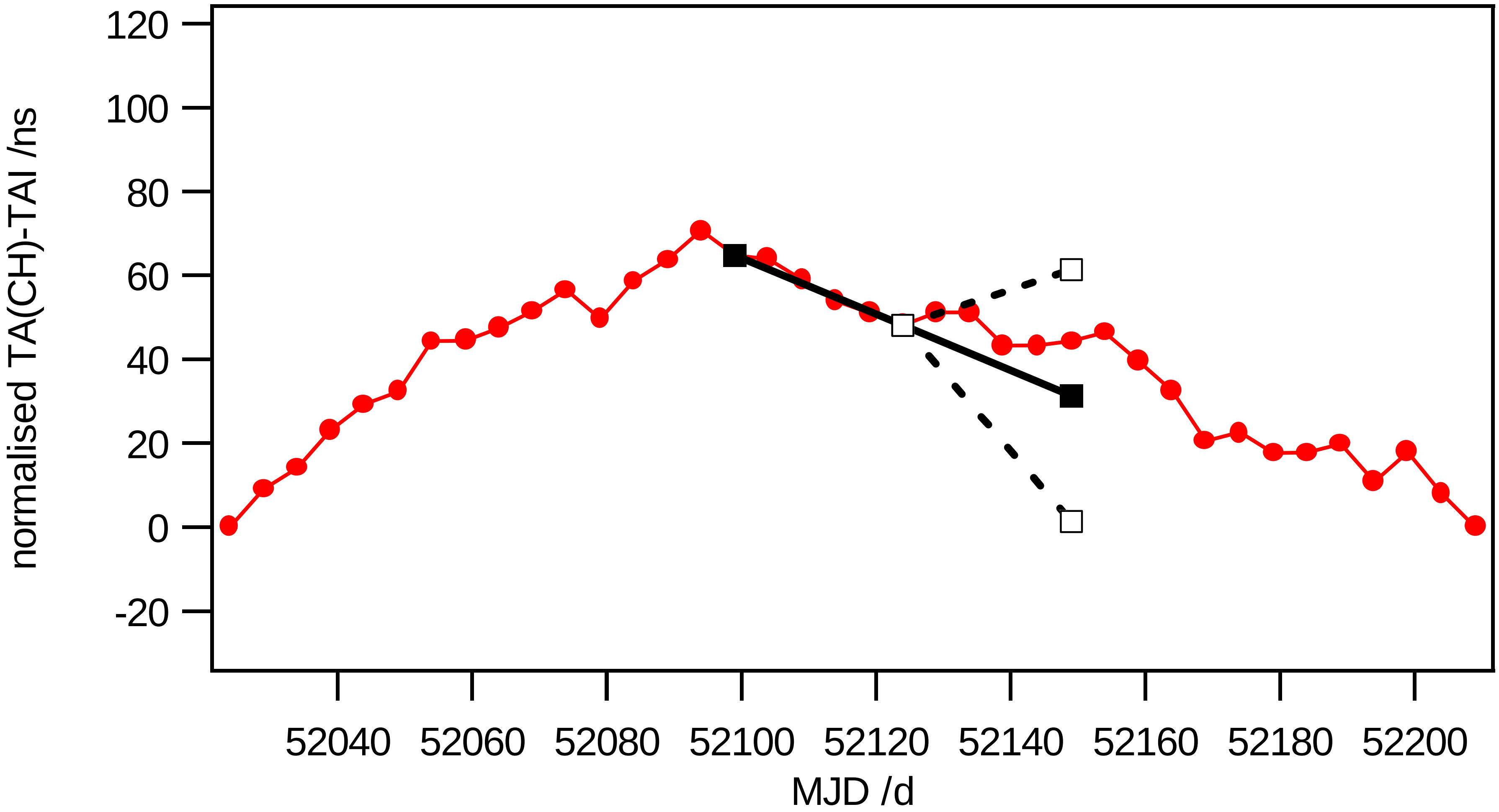


Metrology of Clocks and Timescales - METAS 2002-04-24

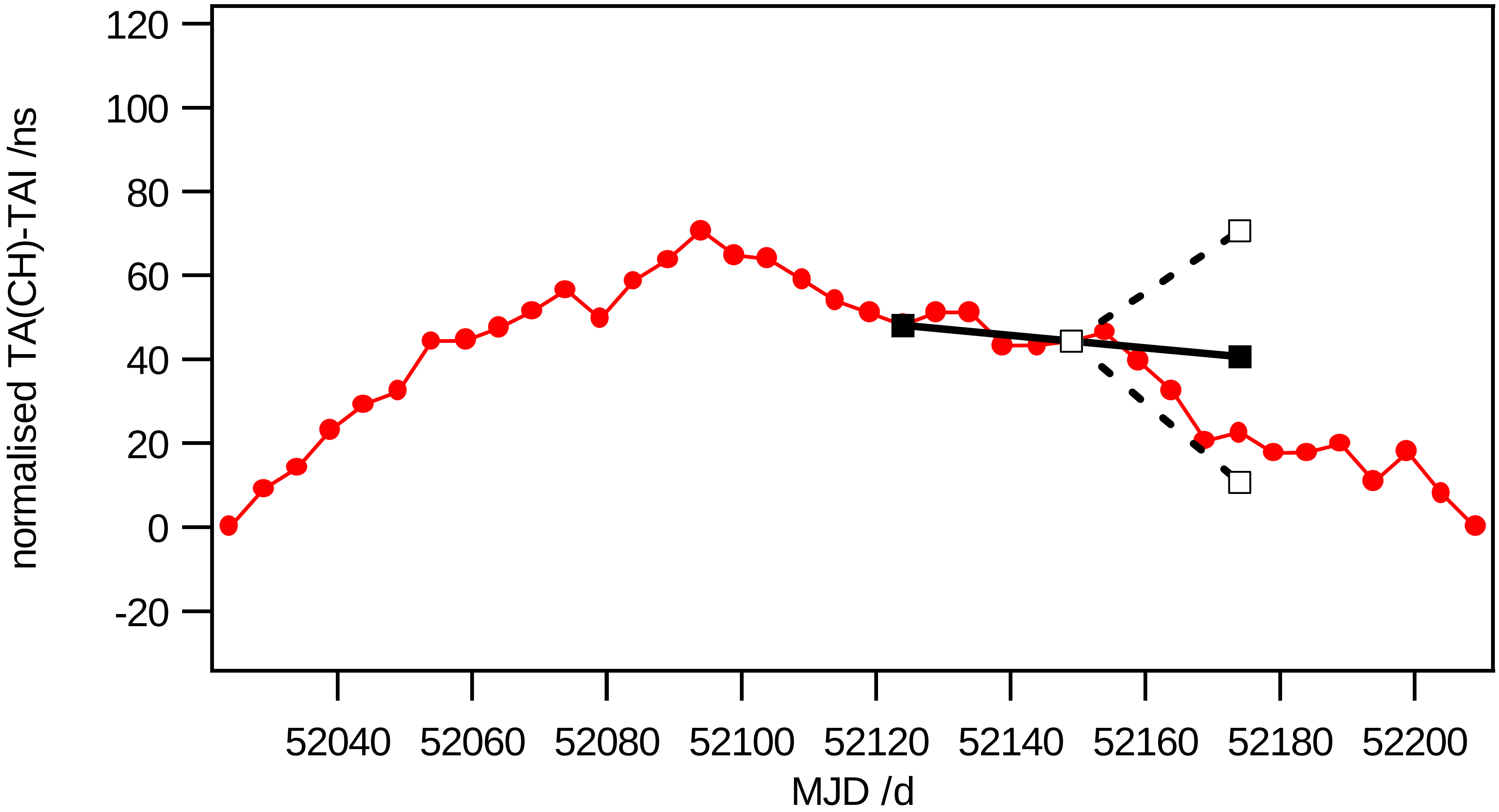
linear prediction : example



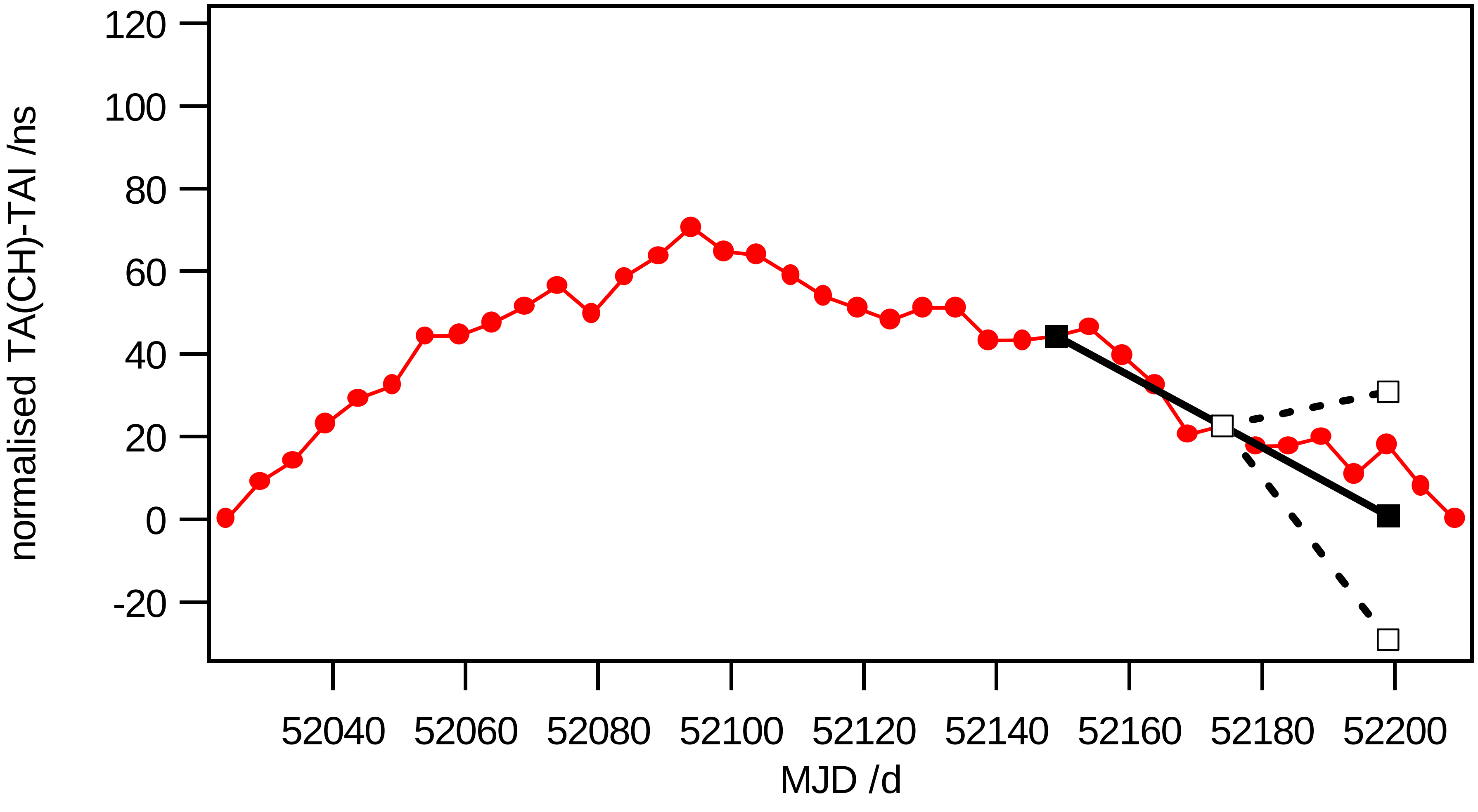
linear prediction : example



linear prediction : example



linear prediction : example



interpolation of timescales (A)

- ✓ **data acquisition of clock data for TA(CH) and UTC(CH)**
  - ☞ TA(CH) and UTC(CH) are computed daily for 00:00 UTC
  - ☞ but the phase of the clocks is measured every 3 minutes
  - ☞ the measurement of the clocks and the calculation of the timescales must go on 365d a year…

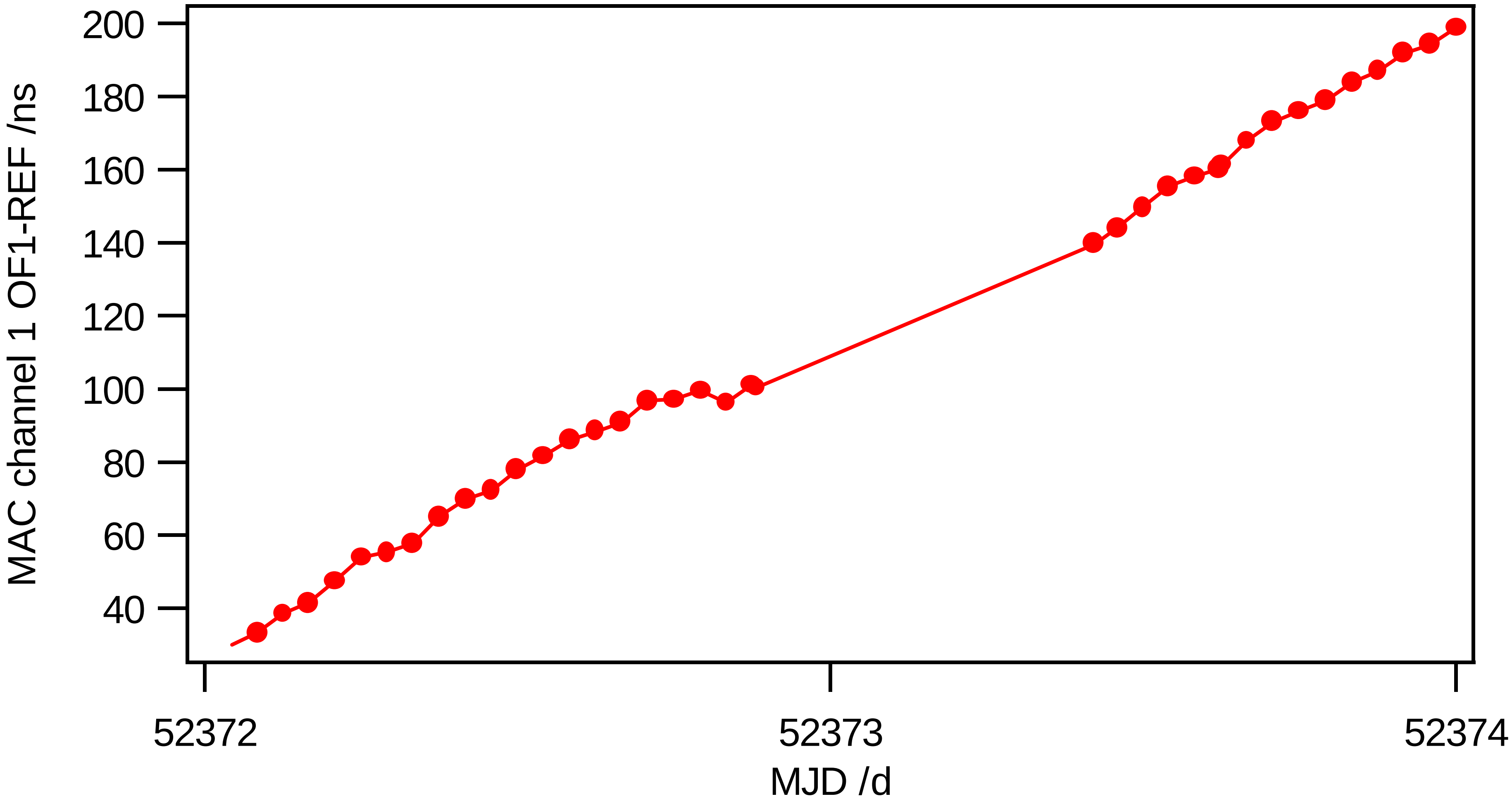
- ✓ **crash of the hard disk on machine time...**
  - ☞ accident on MJD 52373 (09.04.2002) circa 09:00 UTC
  - ☞ last backup performed on MJD 52372, 13h before crash
  - ☞ the 00:00 UTC data for MJD 52373 is missing
  - ☞ interpolation of missing data can be vital...

interpolation of timescales (B)

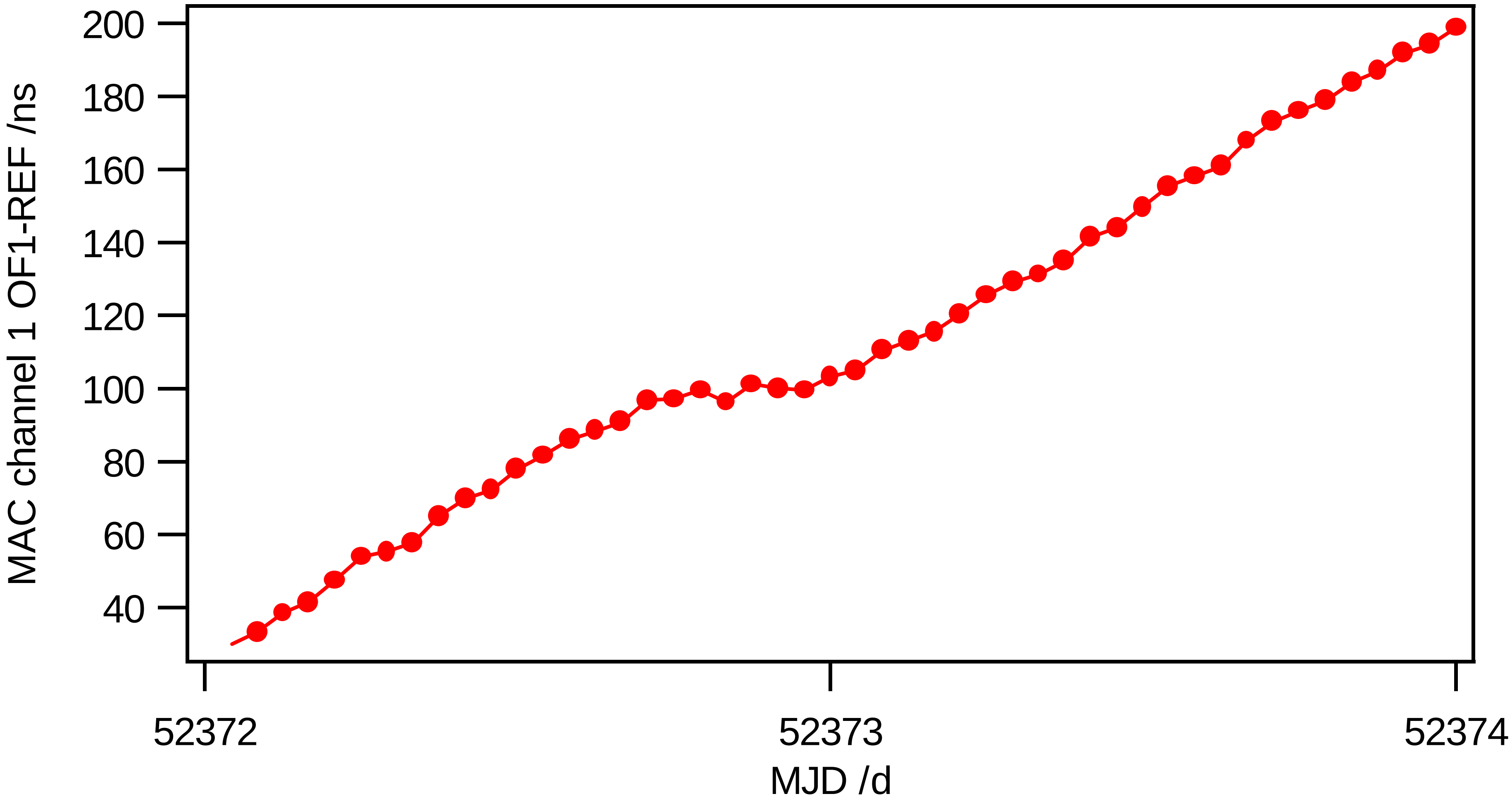
✓ **stochastic interpolation of missing data**

- ☞ linear prediction works like a mirror image with respect to the plane defining the present
- ☞ the mirror process has the same statistical properties as the original
- ☞ the **data segment** before the blank is **predicted forward**
- ☞ the **data segment** after the blank is **predicted backward**
- ☞ if **the two predictions are averaged**, continuity is preserved and the statistical properties are preserved too

interpolation : before...



interpolation : after...



uncertainty on timescale difference calibrations

- ✓ **case of post-processed calibration vs UTC(CH)**
  - ☞ local comparison with REF by Time Interval Counter
  - ☞ uncertainty limited by measurement noise (1 ns)
  - ☞ uncertainty limited by calibration of delay in cables (2 ns)
- ✓ **case of post-processed calibration vs UTC**
  - ☞ uncertainty limited by GPS common view comparison
  - ☞ precision : **1 ns** (1 day average, <1000 km baseline)
  - ☞ accuracy : **10 ns** (depends on **definition of reference plane)**
- ✓ **case of real-time calibration vs UTC(CH) or UTC**
  - ☞ uncertainty limited by **error on prediction**
  - ☞ prediction of UTC(CH) vs UTC over 1 month (30 ns)
  - ☞ prediction of REF - UTC(CH) over 48h (7 ns)

uncertainty on frequency calibrations (A)

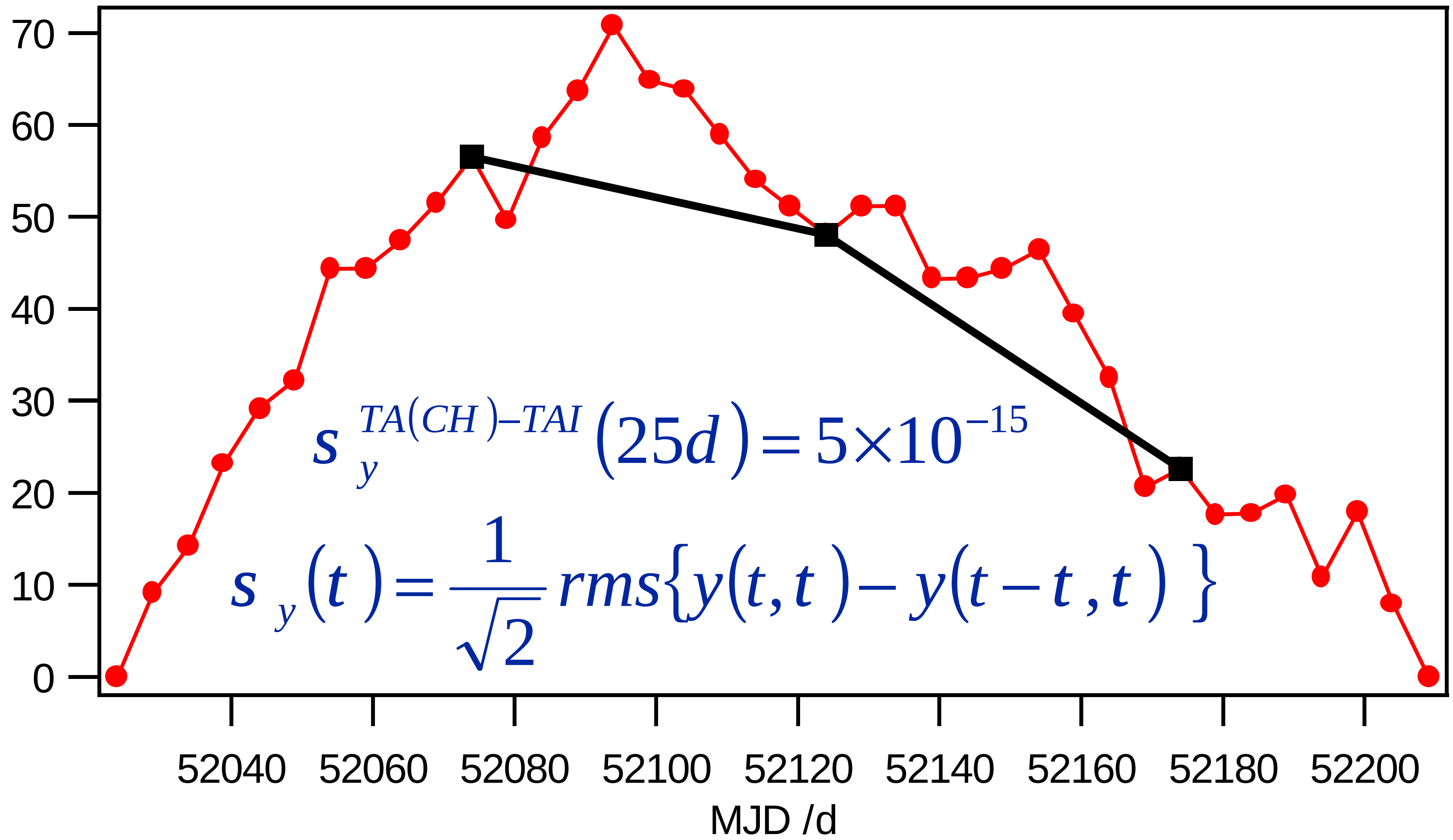
- ✔ **comparison with a cesium primary standard**
  - ☞ SI **s** unit defined as the duration of **9 192 631 770 periods** of the **cesium hyperfine resonance**
  - ☞ in **cesium fountains** the atoms are cooled by laser beam
  - ☞ accuracy of **1×10^{-15}** can be achieved
  - ☞ METAS does not have yet a primary standard…
  - ☞ ...wait until the **end of this year** (development with ON)
- ✔ **comparison with UTC(CH) or UTC**
  - ☞ uncertainty limited by the stability of the DUT
  - ☞ the **Allan deviation** of DUT-UTC on the averaging interval yields the uncertainty on the frequency comparison
  - ☞ the optimum averaging interval minimises the Allan deviation

uncertainty on frequency calibrations (B)

$$s_y(t) = \frac{1}{\sqrt{2}} rms\{y(t,t) - y(t-t,t)\}$$

- ✓ the rms change of slope involves **a pair** of measurements
- ✓ dividing by $\sqrt{2}$ yields uncertainty on a single measurement
- ✓ thus the **Allan deviation** gives directly the **uncertainty** on a **single measurement**
- ✓ uncertainty includes both the averaging of the **additive noise** and the **actual non-stationary change of the mean value**

uncertainty on frequency calibrations (C)



$$s_y^{TA(CH)-TAI}(25d) = 5\times 10^{-15}$$

$$s_y(t) = \frac{1}{\sqrt{2}} rms\{y(t,t) - y(t-t,t)\}$$

conclusion : application of T&F tools to other fields of metrology

- ✓ the Allan deviation is no longer considered as an exotic statistical tool used exclusively by time & frequency specialists…
- ✓ … because the non stationarity of the noise processes is not an exclusivity of time & frequency metrology
- ✓ more and more, the Allan deviation is applied to other fields of metrology
- ✓ see example on next slide
- ✓ other tools such as linear prediction or stochatic interpolation might be useful too

Allan deviation applied to electrical measurements (A)

✔ for example :

☞Thomas J. Witt, BIPM, *IEEE Trans. Instrum. Meas.* Vol. 50 No. 2, April 2001, pp. 445-448.

☞**« Using the Allan Variance and Power Spectral Density to Characterize DC Nanovoltmeters »**

Allan deviation applied to electrical measurements (B)

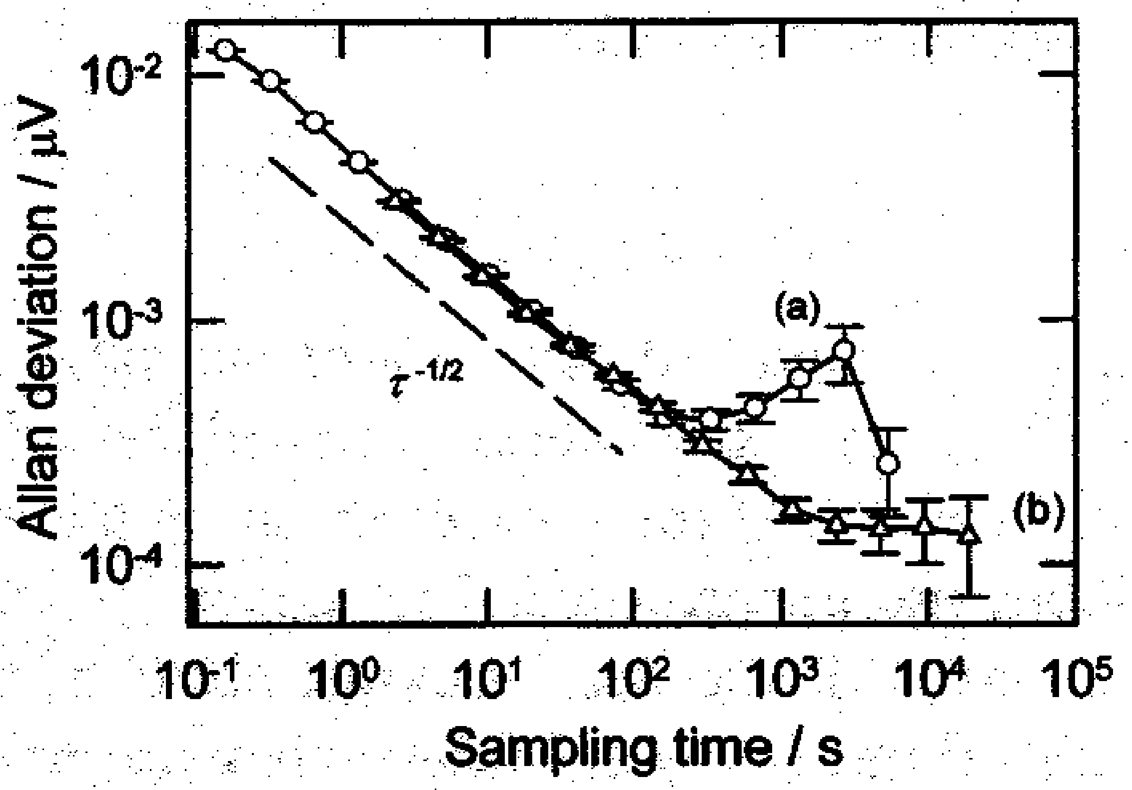


Fig. 1. Allan deviation for a DVM-A (a) at ambient temperature conditions and (b) in temperature-stabilized enclosure.